

Classical Analysis – Math 108b

Homework #8

due 11 am Monday, March 7, 2005.

1. "Sketch" the graph of the the function
 - a) $\sin 10000x$
 - b) $\sin 10000x + \sin 10002x$
2. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.
3. Consider $g(x) = f(x + \alpha)$, $f \in C^{2\pi}$. What is the relationship between the Fourier coefficients of f and g ?
4. Let $f \in C^{2\pi}$ and let $f'(x_0)$ exist. Prove that $s_n(f)(x_0) \rightarrow f(x_0)$ as $n \rightarrow \infty$.
5. Modify the proof of Fejer's Theorem to show that it holds "pointwise". That is, let $f \in L_2[-\pi, \pi]$, let $\sigma_n(f) = \frac{1}{n} \sum_{k=0}^{n-1} s_k(f)$ and let f be continuous at $x \in (-\pi, \pi)$. Prove that $\sigma_n(f)(x) \rightarrow f(x)$ as $n \rightarrow \infty$.