Classical Analysis – Math 108b

Homework # 4

due 11 am Monday, February 7, 2005.

1. If $f \in R[a,b]$ and $f = g$ a.e., does it follow that $g \in R[a,b]$? What if "a.e." is weakened to "except at countably many points"? Or to "except at finitely many points"?

2. Suppose that $f : D \to \mathbb{R}$, where D is measurable. Show that f is measurable if and only if $\{f > \alpha\}$ is measurable for each *rational* α.

3. Exercise 11.3 in [Rudin] (i.e., Exercise 3 in Chapter 11 of Walter Rudin, Principles of Mathematical Analysis, Third edition.)

4. If $f, g \in R[a,b]$ and $f = g$ a.e., does it follow that $\int_a^b f dx = \int_a^b g\, dx$?

5. Let E be a measurable subset of $\mathbb{R}^1$ such that $m(E) > 0$. Prove that for every $0 < \varepsilon < 1$ there exists a bounded open interval I in $\mathbb{R}^1$ such that $\varepsilon m(I) < m(E \cap I)$.