

Classical Analysis – Math 108b

Homework # 3

due 11 am Monday, January 31, 2005.

1. Exercise 6.8 in [Rudin] (i.e., Exercise 8 in Chapter 6 of Walter Rudin, Principles of Mathematical Analysis, Third edition).

2. Given a positive continuous linear functional $L : C[a, b] \rightarrow \mathbb{R}^1$, find an increasing function $\alpha : [a, b] \rightarrow \mathbb{R}^1$ such that $\forall f \in C[a, b] \quad L(f) = \int_a^b f d\alpha$.

a) $L(f) = \frac{1}{2}(f(a) + f(b));$

b) $L(f) = \sum_{n=1}^{\infty} \frac{1}{2^n} f\left(\frac{1}{n}\right);$

c) $L(f) = \int_a^{\frac{b+2a}{3}} f(x)dx + \int_{\frac{2b+a}{3}}^b f(x)dx;$

d) $L(f) = \sum_{n=1}^{\infty} \frac{1}{n} \int_{a+\frac{b-a}{2^n}}^{a+\frac{b-a}{2^{n-1}}} f(x)dx.$

3. Find the length of each of the following curves

a) $\gamma : [0, 1] \rightarrow \mathbb{R}^3, \quad \gamma(t) = (t, \cos 2\pi t, \sin 2\pi t);$

b) $\gamma : [0, 1] \rightarrow \mathbb{R}^2, \quad \gamma(t) = (\cos 2\pi t, \sin 2\pi t);$

c) $\gamma : [0, 1] \rightarrow \mathbb{R}^2, \quad \gamma(t) = (\cos 4\pi t, \sin 4\pi t).$

4. If $f : \mathbb{R}^1 \rightarrow \mathbb{R}^1$ satisfies $|f(x) - f(y)| \leq K|x - y|$ for all x and y , show that $m^*(f(E)) \leq Km^*(E)$ for any $E \subset \mathbb{R}^1$.

5. Let E denote the set of all real numbers in $[0, 1]$ whose decimal expansions contain no 2's or 6's. Prove that $m^*(E) = 0$.