Classical Analysis – Math 108b

Midterm

due 5 pm Tuesday, February 7, 2006 (no other time limits).

1. Show that if $f, g \in R[a,b]$, then $\max(f,g)$ and $\min(f,g)$ are integrable on $[a,b]$, and

$$\int_a^b \max(f(x), g(x)) \geq \max\left(\int_a^b f(x)dx, \int_a^b g(x)dx\right) \quad \text{and}$$

$$\int_a^b \min(f(x), g(x)) \leq \min\left(\int_a^b f(x)dx, \int_a^b g(x)dx\right).$$

2. Let G be an open set containing the rationals in $[0,1]$ with $m(G) < \frac{1}{2}$. Prove that χ_G is not Riemann integrable on $[0,1]$, and is not equal a.e. to any Riemann integrable function on $[0,1]$.

3. Prove that $E \subset [0,1]$ is measurable if and only if $m^*(E) + m^*([0,1]\backslash E) = 1$.

4. If $m^*(E) > 0$ for a set $E \subset \mathbb{R}^1$, show that E contains a non-measurable subset.

5. Suppose $E \subset \mathbb{R}^1$ is measurable and $m(E) = 1$. Prove that for every $\alpha \in (0,1)$ there exists a compact set $F_\alpha \subset E$, $m(F_\alpha) = \alpha$, consisting entirely of irrationals.