Classical Analysis – Math 108b

Homework #7
due 11 am Monday, March 6, 2006.

1.(10) Exercise 15.1 in [Carothers].

2.(10) "Sketch" the graph of the the function

a) $\sin 10000x$

b) $\sin 10000x + \sin 10002x$

3.(10) Exercise 15.2 in [Carothers].

4.(10) Consider $g(x) = f(x+\alpha)$, $f \in C^{2\pi}$. What is the relationship between the Fourier coefficients of f and g?

5.(10) Prove that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

Complementary problems:

6.(5) Let $f : \mathbb{R} \to \mathbb{R}$ be a 2π-periodic C^2 function. Prove that there exists $M > 0$ such that for every $n \in \mathbb{N}$ and every $x \in \mathbb{R}$

$$|s_n(f)(x) - f(x)| \leq \frac{M}{n}.$$

7.(5) Define the Legendre polynomials $\{P_n\}$ by $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$.

(a) Compute the first few Lengendre polynomials, and compare with what you get by Gram-Schmidt orthogonalization of the monomials $1, x, x^2, \ldots$ in $L_2([-1,1])$.

(b) Show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$.

(c) Prove that Legendre polynomials form an orthogonal basis of $L_2([-1,1])$. Suppose that $f \in L_2([-1,1])$ is given by $f(x) = \sum_{n=1}^{\infty} c_n P_n(x)$. Compute c_n and say explicitly in what sense the series converges.