An automated model reduction tool to guide the design and analysis of synthetic biological circuits

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Motivation

A reduced order model is

- a lower dimensional model that has a simple representation,
- computationally efficient, and
- easier to use for system design compared to any other higher order, more complex mathematical model.

We aim to use a reduced model to

- better understanding of key processes,
- improving parameter identifiability, and
- guide experimental design.

Results

Problem Formulation

Mathematical model

\[ \dot{x} = f(x, \theta) \]

\[ y = \xi x \]

Algorithm

Model input

<table>
<thead>
<tr>
<th>Full Model</th>
<th>Reduced Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear dynamics</td>
<td>Nonlinear dynamics</td>
</tr>
<tr>
<td>n states (x)</td>
<td>r states (x)</td>
</tr>
<tr>
<td>p outputs (y)</td>
<td>q outputs (y)</td>
</tr>
</tbody>
</table>

Solution

Post reduction analysis

<table>
<thead>
<tr>
<th>Reduced model 1</th>
<th>Reduced model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced Model 1</td>
<td>Reduced Model 2</td>
</tr>
</tbody>
</table>

Theoretical results

From [1], we have,

\[ \max_{\lambda_{H}} \| p \|^2 = \max \{ \lambda_{H} \} \]

\[ Q = \left[ \begin{array}{cccc} T_1 P_{11} T_1 + T_2 P_{21} T_1 + P_{12} \ T_2 P_{12} + P_{22} T_2 \end{array} \right] \]

Our work extends this to include bounds on sensitivity of error for both linear and nonlinear dynamics. The full biofilm paper for our work is available at [2].

References


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