1. Prove that $\operatorname{Spec} R$ is irreducible and reduced if and only if R is an integral domain.

2. Let G be an abelian group. Describe the sheaf associated to the constant presheaf $U \mapsto G$.

3. Let $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{O}$ be sheaves on a topological space. Show by an example in each case that the presheaves given below are not sheaves in general.

   a) $U \mapsto \operatorname{im} \theta_U$, where $\theta : \mathcal{F} \to \mathcal{G}$ is a morphism.

   b) $U \mapsto \mathcal{F}(U)/\mathcal{G}(U)$, where $\mathcal{G}$ is a subsheaf of $\mathcal{F}$.

   c) $U \mapsto \mathcal{F}(U) \otimes_{\mathcal{O}(U)} \mathcal{G}(U)$, where $\mathcal{F}$ and $\mathcal{G}$ are $\mathcal{O}$-modules.

4. If $\mathcal{G}$ is a subsheaf of a sheaf $\mathcal{F}$ prove that we have the following sheaf-exact sequence where all the morphisms are defined naturally:

$$0 \to \mathcal{G} \to \mathcal{F} \to \mathcal{F}/\mathcal{G} \to 0.$$

5. If $\phi : \mathcal{F} \to \mathcal{G}$ is a morphism of sheaves then prove

   a) $\operatorname{im} \phi \cong \mathcal{F}/\ker \phi$.

   b) $\operatorname{coker} \phi \cong \mathcal{G}/\operatorname{im} \phi$.