1. Prove that $\operatorname{Spec} A$ is a T_0-space for any ring A.

2. Let $\phi : A \to B$ be a homomorphism of rings and let $f \in A$. Then prove the following:

   i) $\operatorname{Im}(\phi^\flat) \subseteq V(f) \Leftrightarrow \phi(f)$ is nilpotent.

   ii) $\operatorname{Im}(\phi^\flat) \cap V(f) = \emptyset \Leftrightarrow \phi(f)$ is a unit.

3. Let $X = \operatorname{Spec} A$ for some ring A and let U be an open subset of X containing $x_1, \ldots, x_r \in X$. Show that there exists $f \in A$ satisfying

$$\{x_1, \ldots, x_r\} \subseteq D(f) \subseteq U.$$

4. Let k be an algebraically closed field of characteristic 0. Consider the map

$$\operatorname{Spec} k[[x, y]] \to \operatorname{Spec} k[x, y]$$

   associated to the natural homomorphism $k[x, y] \to k[[x, y]]$.

   i) Let $u = \sqrt{x^2 + x^3} = x + \frac{1}{2}x^2 - \frac{1}{8}x^3 + \cdots$. What is the image of the point $(y - u) \in \operatorname{Spec} k[[x, y]]$ in $\operatorname{Spec} k[x, y]$?

   ii) Show that the image of $(y - \sum_{n \geq 1} x^n/n!)$ is the generic point of $\operatorname{Spec} k[x, y]$.

5. Describe the points and closed sets of $X = \operatorname{Spec} \mathbb{Z}[x]$. What is the dimension of X? Describe the fibers of the natural map

$$\operatorname{Spec} \mathbb{Z}[x] \to \operatorname{Spec} \mathbb{Z}.$$