1. Let $0 \to J_* \to K_* \to L_* \to 0$ be a short exact sequence of chain complexes. Assume $K_n = 0$ for $n < 0$. Use the spectral sequence of filtration to show that there is a long exact sequence of homology groups

$$\cdots \to H_i(J_*) \to H_i(K_*) \to H_i(L_*) \to H_{i-1}(J_*) \to \cdots .$$

2. Use the fibration

$$SO(3) \hookrightarrow SO(4) \to S^3$$

and Leray spectral sequence to compute the homology groups with $\mathbb{Z}$-coefficients of $SO(4)$.

3. If λ is a real line bundle over B and $H^2(B, \mathbb{Z})_{\text{tor}} = 0$ then $\lambda_{\mathbb{C}}$ (the complexification of λ) is a trivial complex line bundle.

4. Let ξ be a rank n real vector bundle given by data $\{(U_\alpha, g_{\alpha\beta})\}$.

   a) Show that $\bigwedge^n \xi$ is determined by $\{(U_\alpha, \det(g_{\alpha\beta}))\}$.

   b) ξ is oriented if and only if $w_1(\xi) = 0$.

5. Suppose $\xi : E \to B$ is a rank 3 complex vector bundle and $\mathbb{P}(\xi) : \mathbb{P}(E) \to B$ is its projectivization. Denote the projection map of $\mathbb{P}(\xi)$ by π. Let η be the quotient bundle of $\pi^*\xi$ by the tautological bundle of $\mathbb{P}(\xi)$. Compute Chern classes of $\mathrm{Sym}^2(\eta^\vee)$ in terms of the Chern classes of ξ and the tautological bundle (assume B is paracompact).