1. Show that a principal bundle is trivial if and only if it admits a section.

2. Show that if ξ and η are respectively a vector bundle and a line bundle over a base B then
$$\mathbb{P}(\xi \otimes \eta) \cong \mathbb{P}(\xi).$$

3. Show for two complex vector bundles ξ and η over a base B
   - $\mathrm{ch}_t(\xi \oplus \eta) = \mathrm{ch}_t(\xi) + \mathrm{ch}_t(\eta)$
   - $\mathrm{ch}_t(\xi \otimes \eta) = \mathrm{ch}_t(\xi)\,\mathrm{ch}_t(\eta)$.

4. Let ξ and η be respectively a holomorphic vector bundle and a holomorphic line bundle over a compact complex manifold X of dimension n. Show that $\chi(\xi \otimes \eta^k)$ is a polynomial in k of degree at most n (here $\eta^k = \eta \otimes \cdots \otimes \eta$). What are the coefficients of k^n, k^{n-1} and the constant term of this polynomial.

5. Let C be a compact Riemann surface of genus g. Let ξ be a holomorphic vector bundle over C of rank r and degree $d =< c_1(\xi), \mu_C >$. Prove the Riemann-Roch formula
$$\chi(\xi) = d + r(1 - g).$$

6. Let $E \to X$ be a complex vector bundle over a compact base. Prove that there exists a continuous map $f : E \to \mathbb{C}^N$ for some N which is injective and linear on fibers. Using this show that any element of $K(X)$ can be written as $\xi - \epsilon$ where ξ is a vector bundle and ϵ is a trivial bundle.

7. Prove that there is a natural ring isomorphism
$$K(X \times S^2) \to K(X) \otimes K(S^2).$$