# AE/AM/CE/ME 102b Homework 1

Mechanics of Structures and Solids - Winter 2012

Due: January 17, 2012 9:00am in class

**Office Hours:**

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1. Consider a body occupying a cylinder of radius $R$ and length $L$, $\Omega = \{(x_1, x_2, x_3) : x_1^2 + x_2^2 < R, 0 < x_3 < L\}$ in the reference configuration. It undergoes the deformation:

$$y_1 = x_1 \cos(\tau x_3) - x_2 \sin(\tau x_3) \quad (1)$$
$$y_2 = x_1 \sin(\tau x_3) + x_2 \cos(\tau x_3) \quad (2)$$
$$y_3 = x_3 \quad (3)$$

where $\tau$ is a constant parameter with unit $[\frac{1}{\text{length}}]$.

a) Describe the deformation. What is the meaning of $\tau$?

b) Find the deformation gradient $F$.

c) Calculate the right Cauchy-Green stretch tensor $C = F^T F$.

2. Consider a square bar occupying the region $(0, L) \times (0, t) \times (0, t)$ in the reference configuration and made of a Mooney-Rivlin material: $\{T = -pI + 2\alpha_1 B - 2\alpha_2 B^{-1}, B = FF^T\}$, subjected to a uniaxial tension with total force $f$ on the faces $\{x_1 = 0\}$ and $\{x_1 = L\}$.

a) Find the relation between the applied force $f$ and the stretch $\lambda_1$ in the $x_1$ direction.

b) Find the Piola-Kirchhoff stress.

c) Find the nominal Poisson's ratio $\frac{\lambda_2 - 1}{\lambda_1 - 1}$ for a small force $f$.

3. Consider a square plate occupying the region $(0, L) \times (0, L), \times (0, t)$ in the reference configuration and made of a Mooney-Rivlin material subjected to a bi-axial stress $\sigma_1$ and $\sigma_2$ as shown in Figure 1. Assume that it undergoes a deformation of the form $\{y_i = \lambda_i x_i, i = 1, 2, 3\}$.

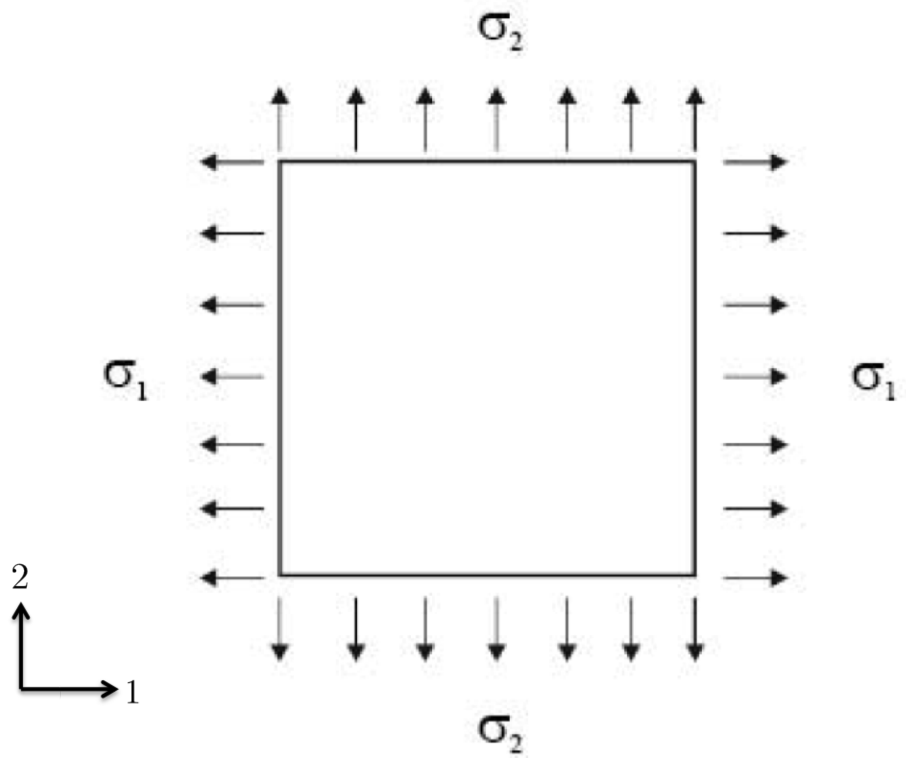


Figure 1: Square plate subjected to biaxial loading (the depth of the plate $t$ is not shown in the picture)

a) Show that diagonal components of the Cauchy Stress tensor $T$ in the 1 and 2 directions are:

$$T_{11} = 2(\lambda_1^2 - \frac{1}{\lambda_1^2\lambda_2^2})(\alpha_1 + \alpha_2\lambda_2^2)$$

$$T_{22} = 2(\lambda_2^2 - \frac{1}{\lambda_1^2\lambda_2^2})(\alpha_1 + \alpha_2\lambda_1^2)$$

b) If the total force $f_1$ on the faces $\{x_1 = \text{constant}\}$ is the same as the total force $f_2$ on the faces $\{x_2 = \text{constant}\}$, does it imply that $\lambda_1 = \lambda_2$? Please explain your reasoning.