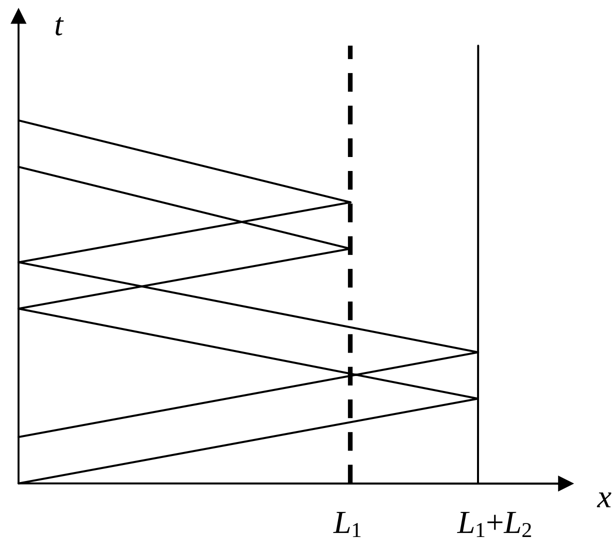
Ae/AM/CE/ME 102c Spring 2007-08
Midterm Examination Solution

Problem 1
(a)



(b) Assuming the bar velocity is C_b, separation occurs after the pulse travels a distance of $3L_1+2L_2$. So $t' = (3L_1+2L_2) / C_b$ and separation first occurs at the junction of the two bars.

(c) The stress on the boundary is

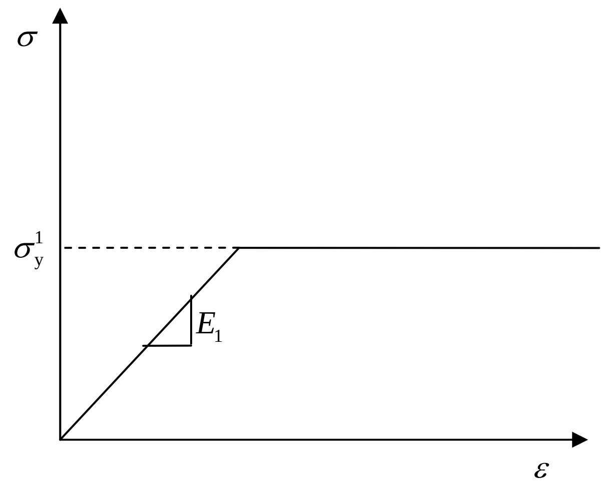
$$\begin{cases} 0 & \text{for } 0 < t < (L_1 + L_2)/C_b = t_1 \\ -2EV_o / C_b & \text{for } t_1 < t < t_1 + T \\ 0 & \text{for } t > t_1 + T \end{cases}$$

Problem 2

For case 1, iso-stress, the effective modulus of the composite is

$$E_{\text{eff}}^{1} = \left(\frac{f_1}{E_1} + \frac{f_2}{E_2} + \frac{f_3}{E_3} \right)^{-1}$$

The stress-strain curve is



For case 2, iso-strain, the effective modulus of the composite is

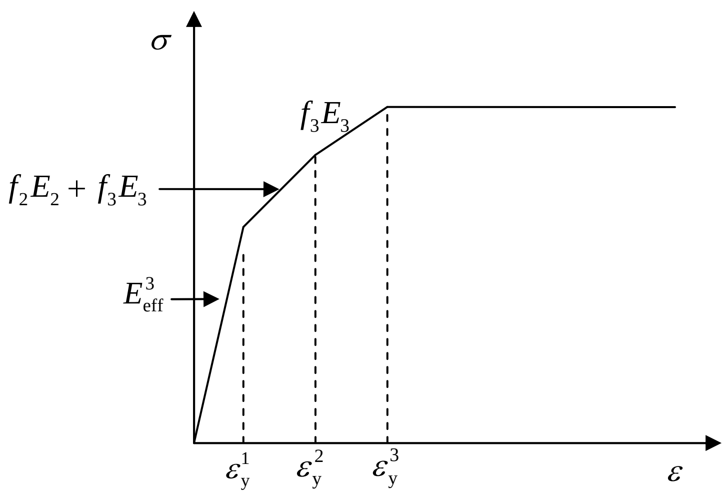
$$E_{\text{eff}}^{2} = f_1E_1 + f_2E_2 + f_3E_3$$

The critical strain $\varepsilon_y^i (i = 1, 2, 3)$ for each material is defined as:

$$\varepsilon_y^i = \frac{\sigma_y^i}{E_i}$$

Without loss of generality, we assume $\varepsilon_y^1 < \varepsilon_y^2 < \varepsilon_y^3$.

The stress-strain curve is



Problem 3

From equilibrium : $\sigma_{11,1} + b_1 = 0$

$$b_1 = +\rho g$$

$$\sigma_{11,1} = -\rho g \rightarrow \sigma_{11} = -\rho g x_1 + c_1$$

$$\sigma_{11} = 0 \ @ \ x = L \rightarrow c_1 = \rho g L$$

$$\sigma_{11} = \rho g (L - x_1)$$

For Kelvin - Voigt : $\varepsilon_D = \varepsilon_S = \varepsilon \ ; \ \sigma_D + \sigma_S = \sigma$

$$\sigma_S = E \varepsilon_S \ ; \ \sigma_D = \eta \dot{\varepsilon}_D$$

$$\sigma_S = E \varepsilon \ ; \ \sigma_D = \eta \dot{\varepsilon}$$

$$E\varepsilon + \eta \dot{\varepsilon} = \sigma = \rho g (L - x_1) \qquad (\tau = \eta / E)$$

$$\dot{\varepsilon} + 1/\tau \, \varepsilon = \frac{\rho g}{E}(L - x_1)$$

$$\varepsilon(t) = \frac{\rho g}{E}(L - x_1)\left(1 - e^{-t/\tau}\right) + c_2$$

$$\varepsilon(0) = 0 \rightarrow c_2 = 0$$

$$\varepsilon(t) = \frac{\partial u(x_1, t)}{\partial x} \quad w/ \quad u(0,t) = 0$$

$$u(x_1, t) = \frac{\rho g}{E}\left(L x_1 - x_1^2/2\right)\left(1 - e^{-t/\tau}\right)$$

$$u(L,t) = \frac{\rho g L^2}{2E}\left(1 - e^{-t/\tau}\right)$$