Hyperbolic geometry of earthquake networks

Ilya Zaliapin[1], Karla Henricksen[1], and Konstantin Zuev[2]

[1]University of Nevada, Reno, Department of Mathematics and Statistics, Reno, United States of America (zal@unr.edu)
[2]California Institute of Technology, Department of Computing + Mathematical Sciences, Pasadena, United States of America (kostia@caltech.edu)

We examine the space-time-magnitude distribution of earthquakes using the Gromov hyperbolic property of metric spaces. The Gromov δ-hyperbolicity quantifies the curvature of a metric space via so-called four-point condition, which is a computationally convenient analog of the famous thin triangle property. We estimate the standard and scaled values of the δ-parameter for the observed earthquakes of Southern California during 1981 – 2017 according to the catalog of Hauksson et al. [2012], the global seismicity according to the NCEDC catalog during 2000 – 2015, and synthetic seismicity produced by the ETAS model with parameters fit for Southern California. In this analysis, a set of earthquakes is represented by a point field in space-time-energy domain D. The Baiesi-Paczuski asymmetric proximity η, which has been shown efficient in applied cluster analysis of natural and human-induced seismicity and acoustic emission experiments, is used to quantify the distances between the earthquakes. The analyses performed in the earthquake space (D,η) and in the corresponding proximity networks show that earthquake field is strongly hyperbolic, i.e. it is characterized by small values of δ. We show that the Baiesi-Paczuski proximity is a natural approximation to a proper hyperbolic metric in the space-time-magnitude domain of earthquakes, with the b-value related to the space curvature. We discuss the hyperbolic properties in terms of the examined earthquake field. The results provide a novel insight into the geometry and dynamics of seismicity and expand the list of natural processes characterized by underlying hyperbolicity.