Entropy and Art
the view beyond Arnheim

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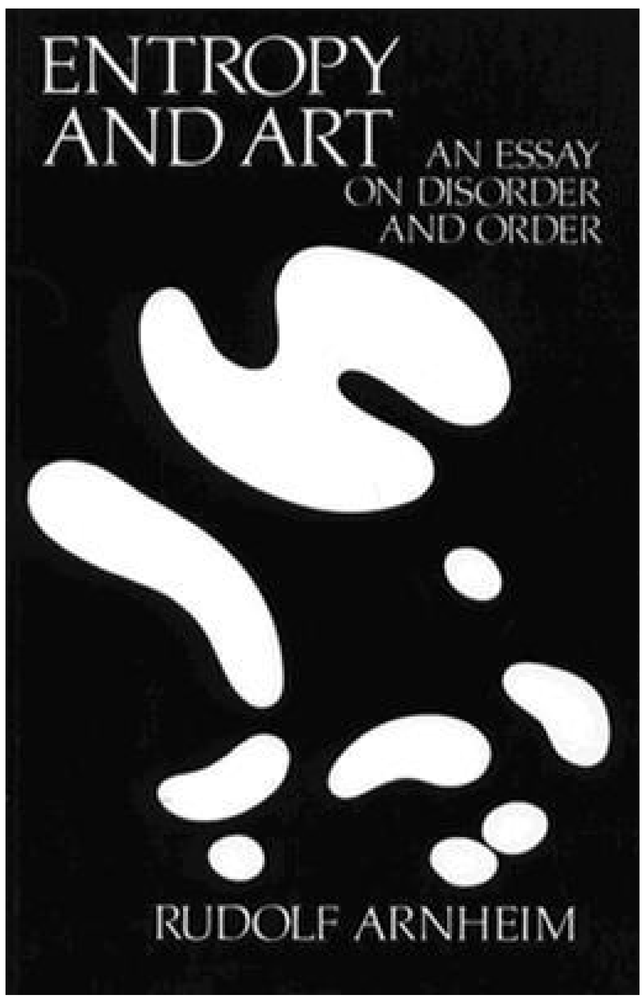
2015

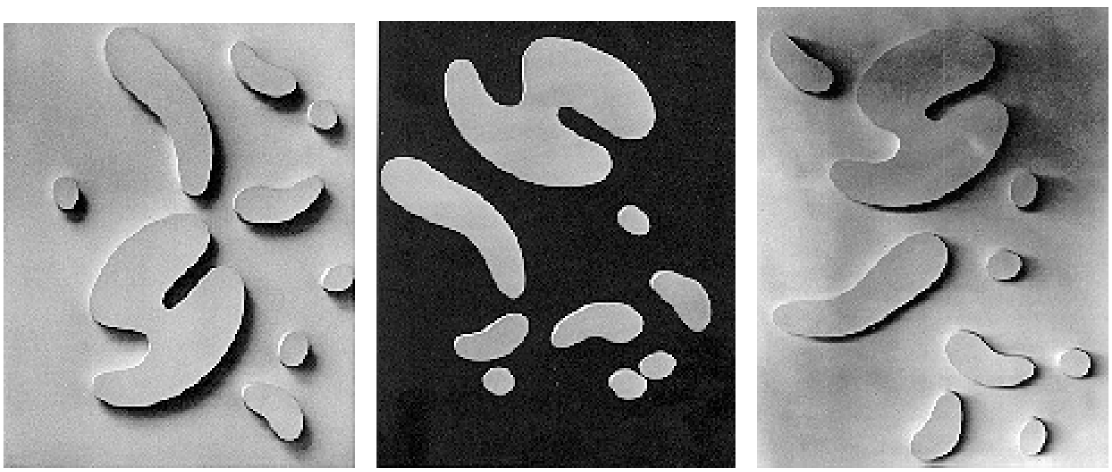
Entropy and Art

• in 1971 art theorist Rudolf Arnheim published one of his most famous essays: "Entropy and Art: an Essay on Disorder and Order"



• Arnheim emphasized the ideas of Entropy = Disorder; Information = Order

• among the examples of art discussed:



Hans Arp, "Three Constellations of Same Forms", 1942

• forms identical and disposed *randomly*: information content in the mutual distances and relative weights of shapes

• randomness is order or disorder? measured by Entropy/Information?

Entropy and Thermodynamics

• the concept of entropy originates in *Thermodynamics*: $\Delta S = \frac{\Delta Q}{T}$

• Laws of Thermodynamics (Energy/Entropy)

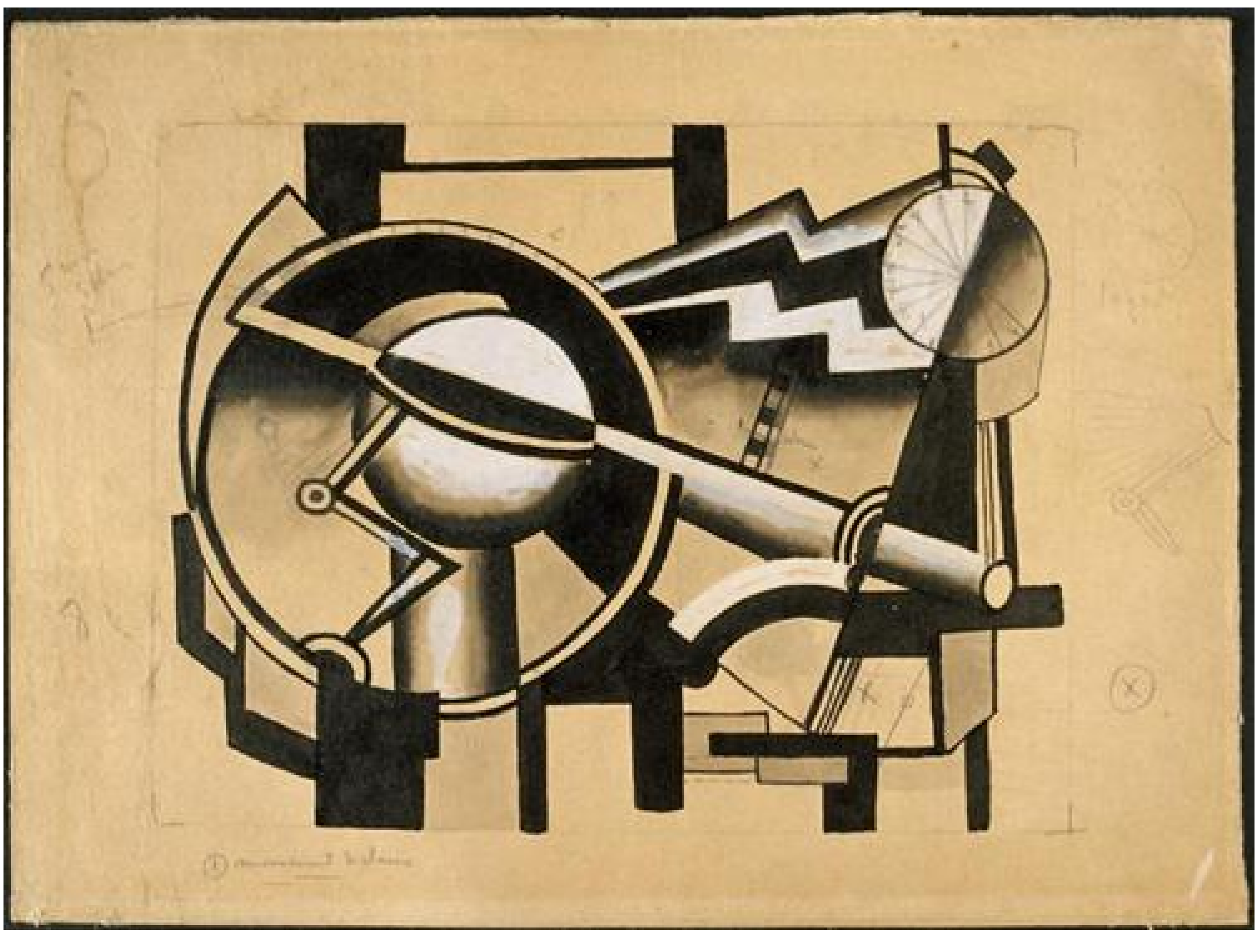
1. Conservation of Energy: the total energy of an isolated system is constant

2. Increase of Entropy: the entropy of an isolated system does not decrease

Entropy measures how much energy degrades towards less and less "usable" forms

... no perpetual motion machine can ever exist

Thermodynamics developed out of the need to understand the efficiency of machines turning energy into work: *Entropy limits the efficiency of mechanical machines*



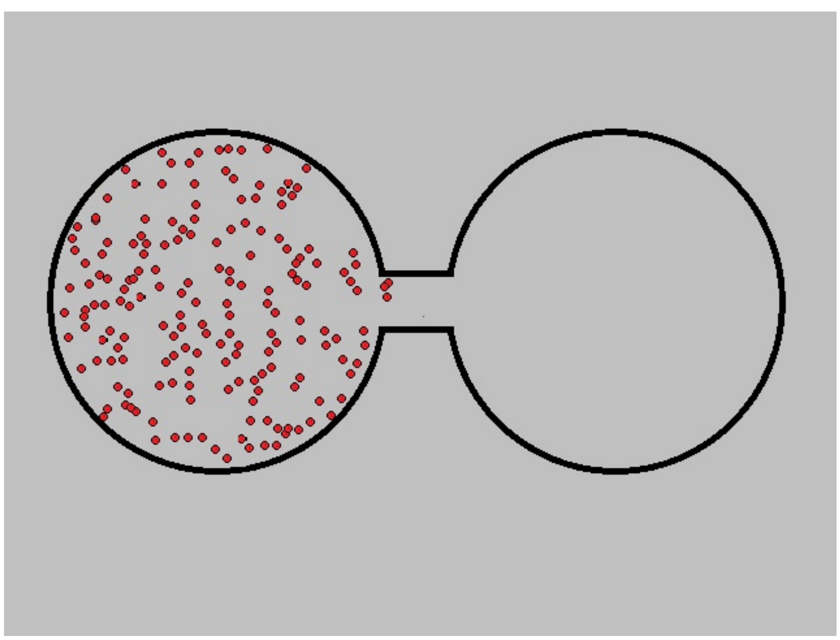
Fernand Leger, "Composition Mécanique (Mouvement de Charrue)"

Entropy and the Arrow of Time

- reversible process $\Delta S = 0$ (entropy stays the same)

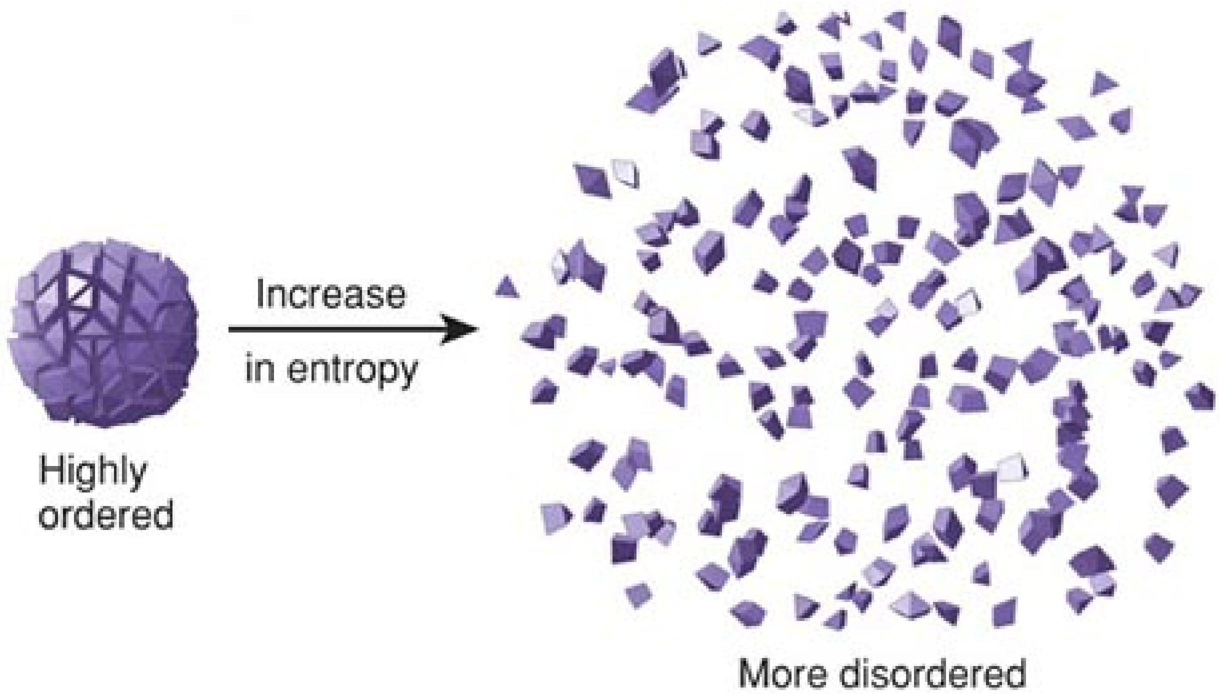
- irreversible process $\Delta S > 0$ (entropy increases)

- *Conclusion*: Entropy is closely related to the *arrow of time*



Anthony Huber, "Entropy IVa", Abstract Constructionism Series, 2014

Entropy as a measure of Disorder



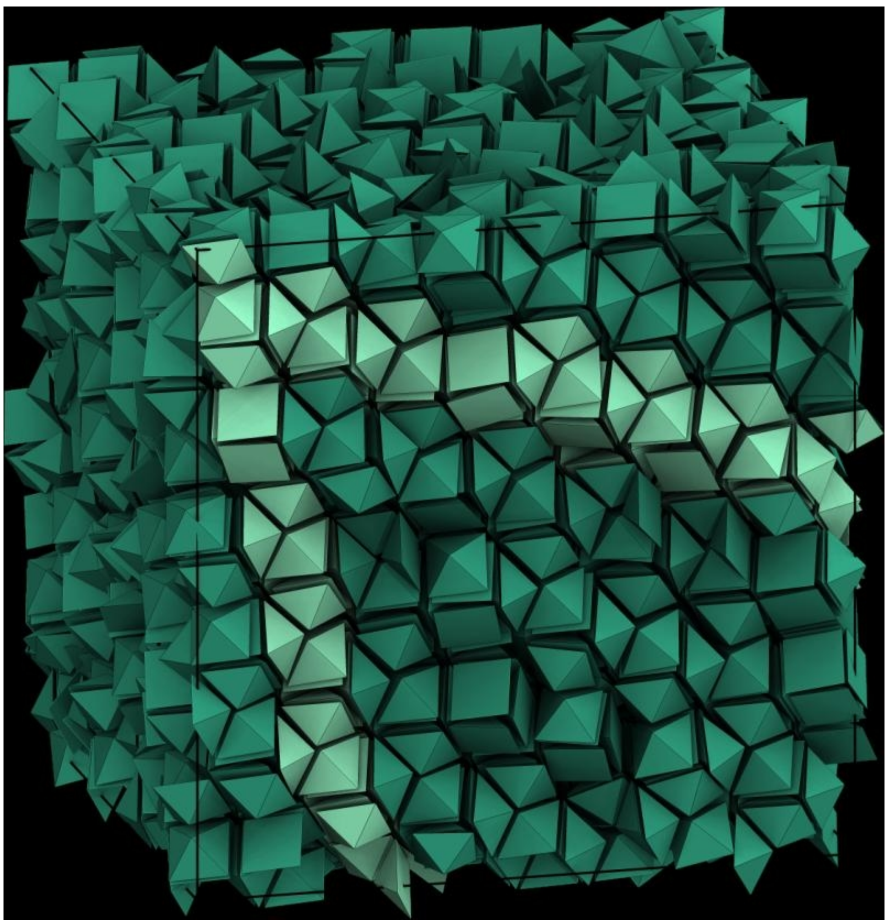
• High Entropy states are disordered; while Low Entropy states (like crystals) have highly ordered structures

Irreversible Transition between Order and Disorder is one of the key concepts in Entropy

Roberto Quintana, "QnTm/FM 2043", Quantum-Foam Series, 2014

Anthony Huber, "Entropy IX", Abstract Constructionism Series, 2014

...**Entropy not always leading to disorder**:
experimentally particles find max entropy arrangement:
if enough space disorder; if crowded crystal structures



P. F. Damasceno, M. Engel, S. C. Glotzer, “Predictive Self-Assembly of Polyhedra into Complex Structures”, Science, 2012; 337 (6093)

Probabilistic description of Thermodynamic Entropy (Ludwig Boltzmann)

• Statistical Mechanics: Entropy measures uncertainty remaining in a system after fixing *macroscopic* properties: temperature, pressure, volume

• coming from *microscopic states* (degrees of freedom) of the system: measures of number of different ways in which a system can be arranged

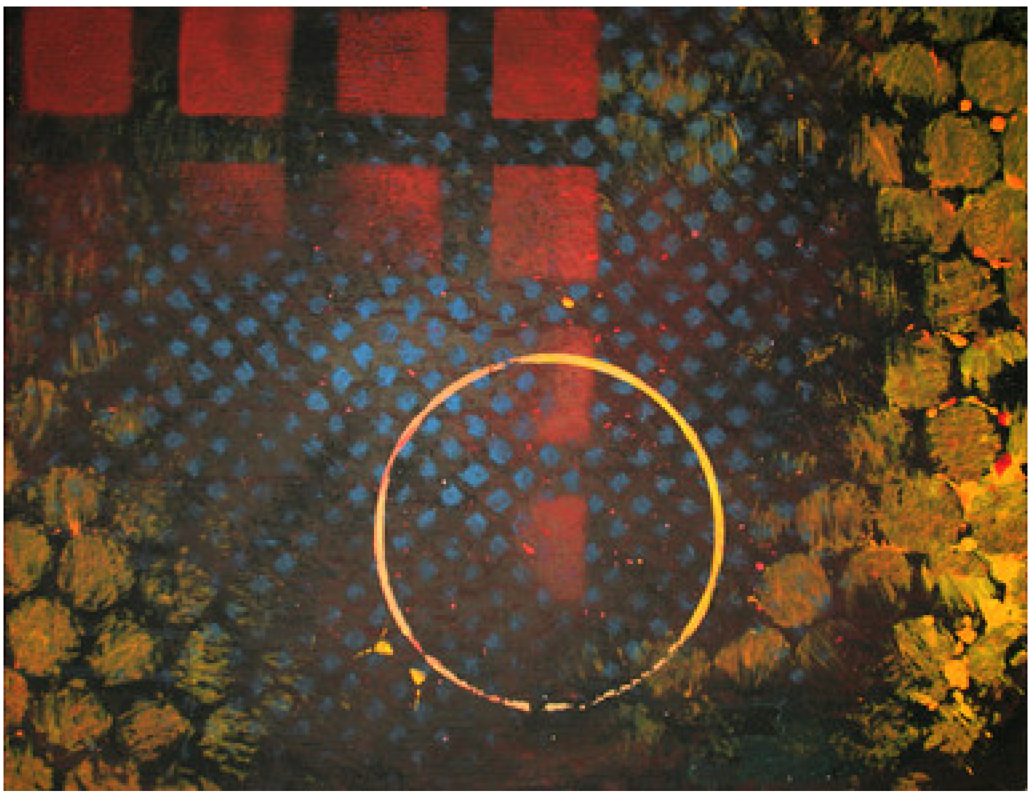
• proportional to the natural logarithm of the number of possible microscopic configurations

$$S = -k_B \sum_i p_i \log p_i$$

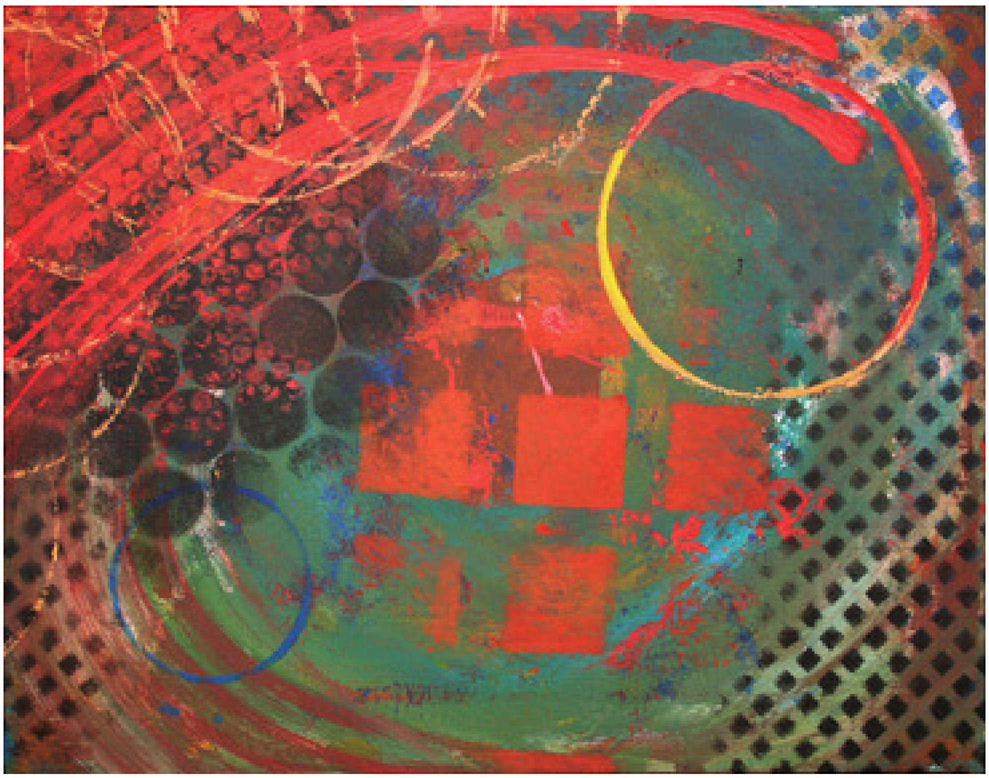
sum over microstates (with probability p_i of finding system in that state

• if all microstates are equally probable
$p_i = 1/N$ and $S = k_B \log N$

Observed properties at macroscopic level (order/disorder) depend on counting all different possible microstates within the macrostate



Roberto Quintana, "QnTm/Fm 207", Quantum-Foam Series, 2014



Roberto Quintana, "QnTm/Fm 230", Quantum-Foam Series, 2014

Gibbs Measure

- if all microstates equally probable $S \sim \log N$

- but probability can depend on *energy*

$$p_i = \frac{e^{-\beta E_i}}{Z(\beta)}$$

energy level E_i; partition function

$$Z(\beta) = \sum_i e^{-\beta E_i} = \text{Tr}(e^{-\beta H})$$

- $\beta = \frac{1}{k_B T}$ inverse temperature: at low temperature probability concentrated on states of lowest energies; at higher temperatures, higher energy states also contribute

- Gibbs measure maximizes the "entropy density" for a fixed "energy density"

Different microscopic states of the system are weighted according to their energies: at low temperature high energy states are inaccessible and the Gibbs measure is concentrated on the ground state; at higher temperatures higher energy states also become accessible

Roberto Quintana, "QnTm/FM 2040", Quantum-Foam Series, 2014

Roberto Quintana, "QnTm/FM 2055", Quantum-Foam Series, 2014

Entropy and Cosmology

• the Universe is an isolated system: by Second Law of Thermodynamics Entropy is increasing

• Big Bang = state of extremely low entropy

• Heat death of the Universe: all the energy reduces to homogeneous distribution of thermal energy

• Gravity causes stars to form and sufficiently massive ones to eventually collapse into black holes

• Black holes have Entropy (Bekenstein–Hawking) equal to the area of their event horizon (maximally entropic states)

• Penrose's Weyl Curvature hypothesis: in a closed universe, Big Bang at very low entropy, while Big Crunch at very high entropy due to black holes: entropy grows like Weyl curvature grows

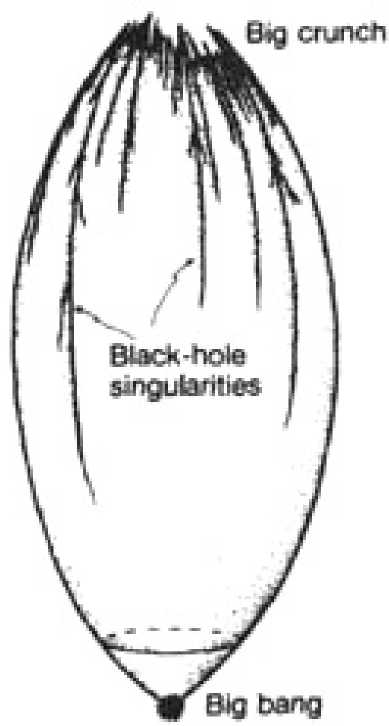
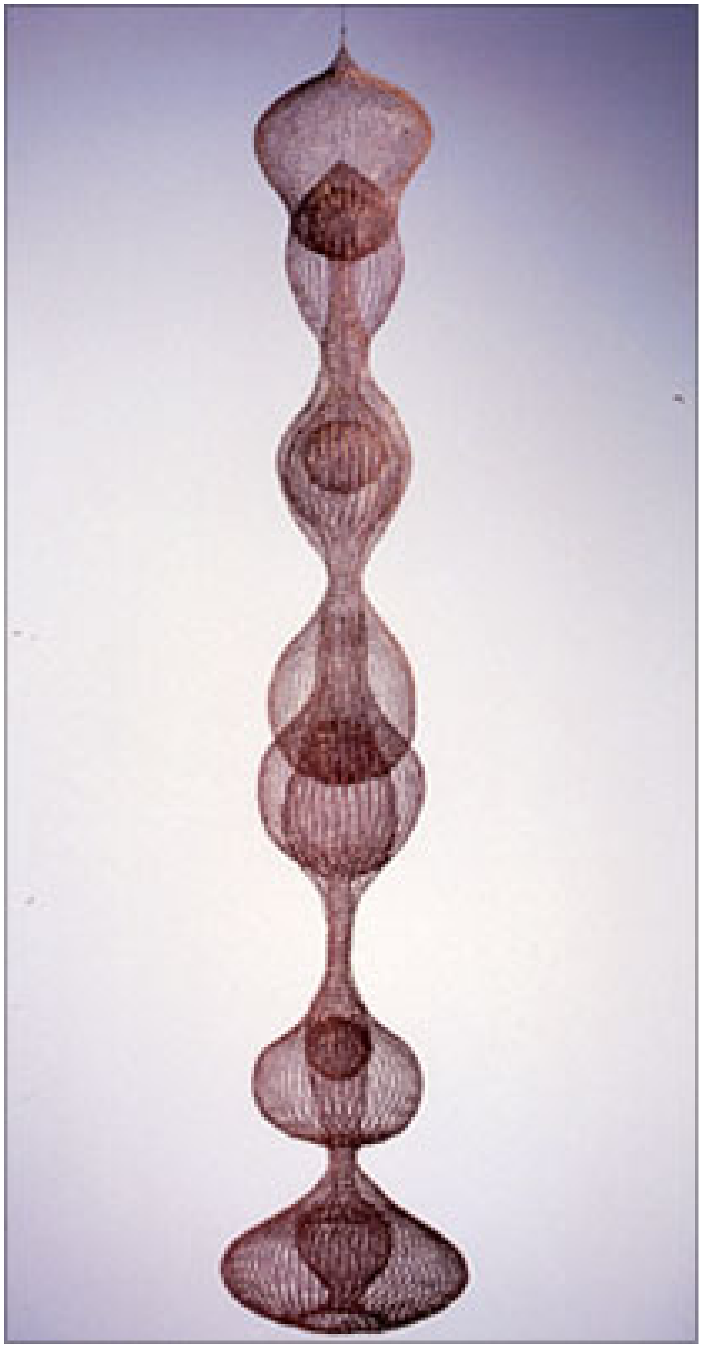


Fig. 7.17. The entire history of a closed universe which starts from a uniform low-entropy big bang, with **WEYL** = 0 and ends with a high-entropy big crunch – representing the congealing of many black holes – with **WEYL** → ∞.

• ... but a lot of problems in applying thermodynamical entropy to cosmology (equilibrium? "entropy gap" in an expanding universe? quantum effects?)

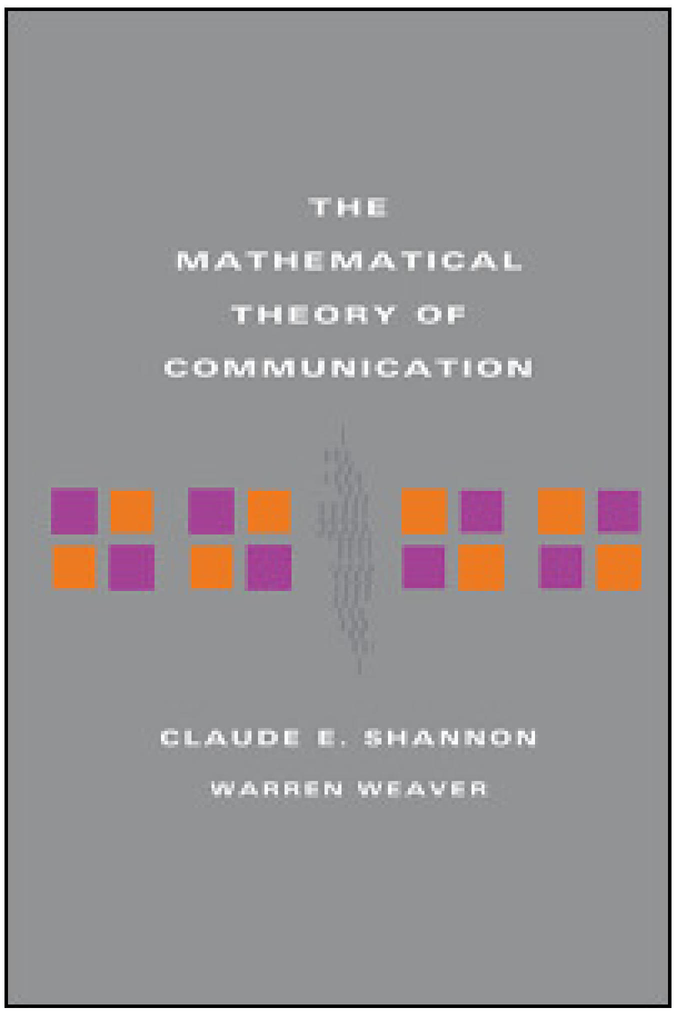
Ruth Asawa, "Untitled, S039", 1959, crocheted wire sculpture



Ruth Asawa, “Untitled, S065”, early 1960s, crocheted wire sculpture

Entropy and Information Theory

• In 1949 mathematician Claude E. Shannon published "The Mathematical Theory of Communication", which introduced the information theoretic meaning of Entropy and ushered the modern era of Information Theory



Entropy and Information

• thermodynamic entropy limits the efficiency of machines (engines)

• information theoretic entropy limits the efficiency of communication

• a signal is encoded, transmitted, and decoded: want to optimize efficiency

• can use short coding for frequently occurring letters, longer for rare ones (optimizing is related to frequencies/probabilities)

• Shannon entropy

$$S = -\sum_i p_i \log(p_i)$$

presence of log function: information is additive over independent events

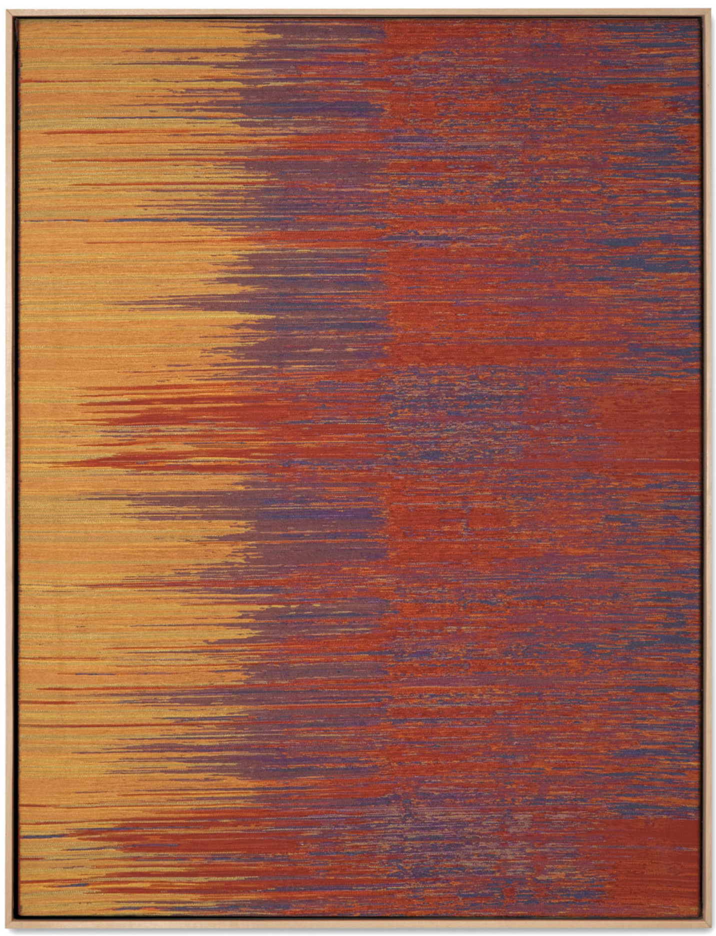
$$\log(p_1 p_2) = \log(p_1) + \log(p_2)$$

• Information = Negative Entropy

• the "Negative Entropy" series of artist Mika Tajima was produced by recording sounds of old Jacquard loom factories in Pennsylvania and turning the sound recording into digital image files

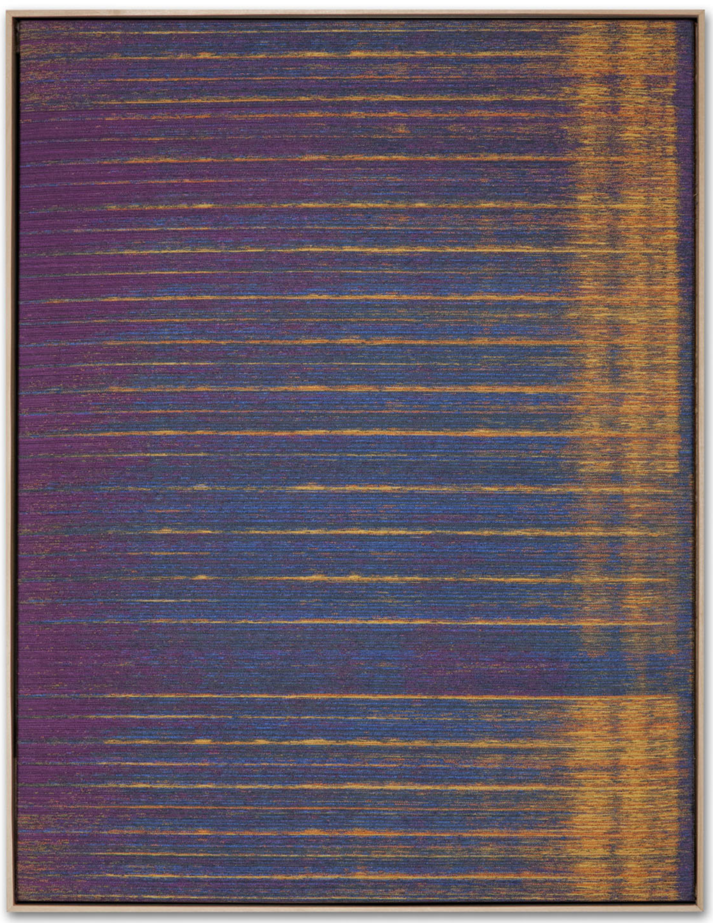
Mika Tajima, "Negative Entropy" 2012

• the programmable Jacquard loom was the historical precursor of the punchcards used to program the first generation of computers in the new “Information Age”



Mika Tajima, “Negative Entropy” 2012

• the connection between Entropy and Information is encoded in the artist's work using the Jacquart loom as a symbol of the transition between the era of mechanical machines (thermodynamics) and the era of information technology and communication



Mika Tajima, "Negative Entropy" 2012

The Park Place Gallery Group

- Group of abstract artists in 1960s New York

- inspiration from Cosmology and General Relativity (explicit theme of the group: "Reimagining Space")

- Also explicit inspiration from Information and Entropy (taken with a grain of salt and a lot of irony)

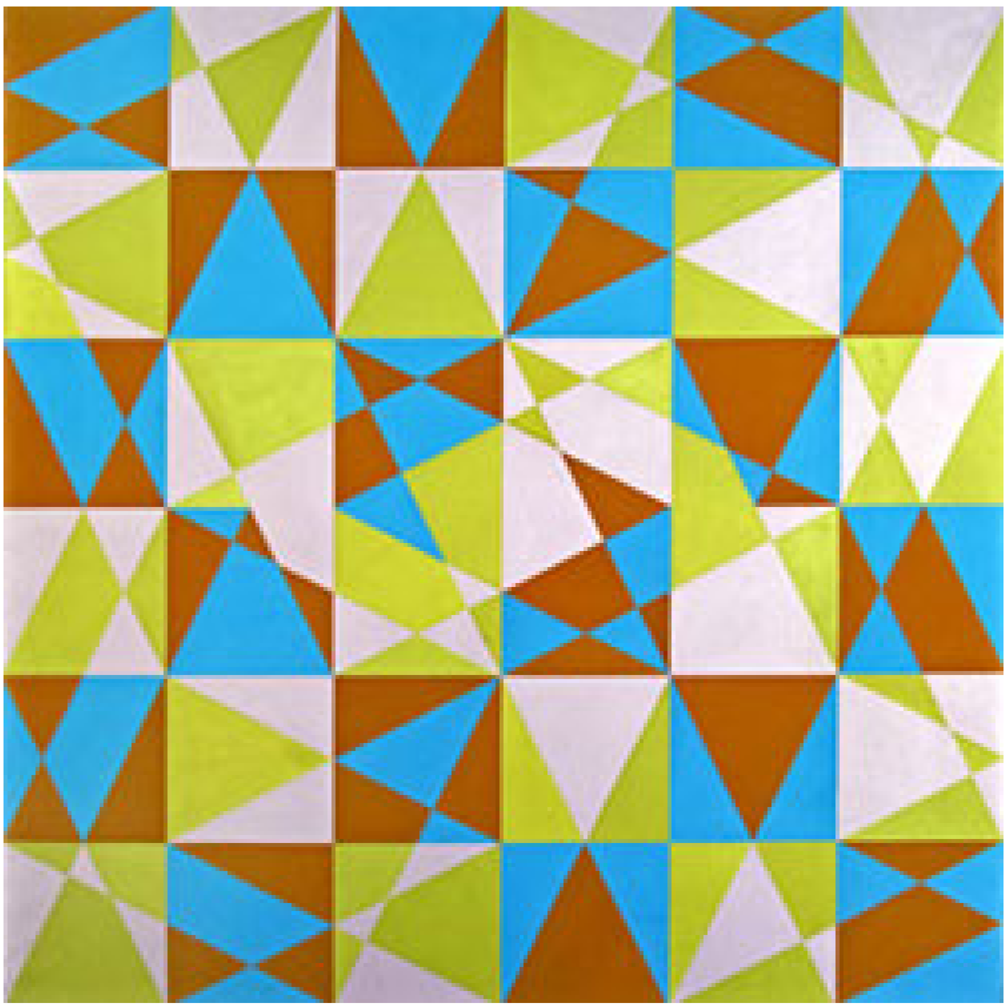
Like energy, entropy is in the first instance a measure of something that happens when one state is transformed into another (P.W. Bridgman, "The Nature Of Thermodynamics")

- Also influenced by Buckminster Fuller's ideas on geometry and synergetics

- An analysis of the role of Entropy in 1960s American Abstract Art:

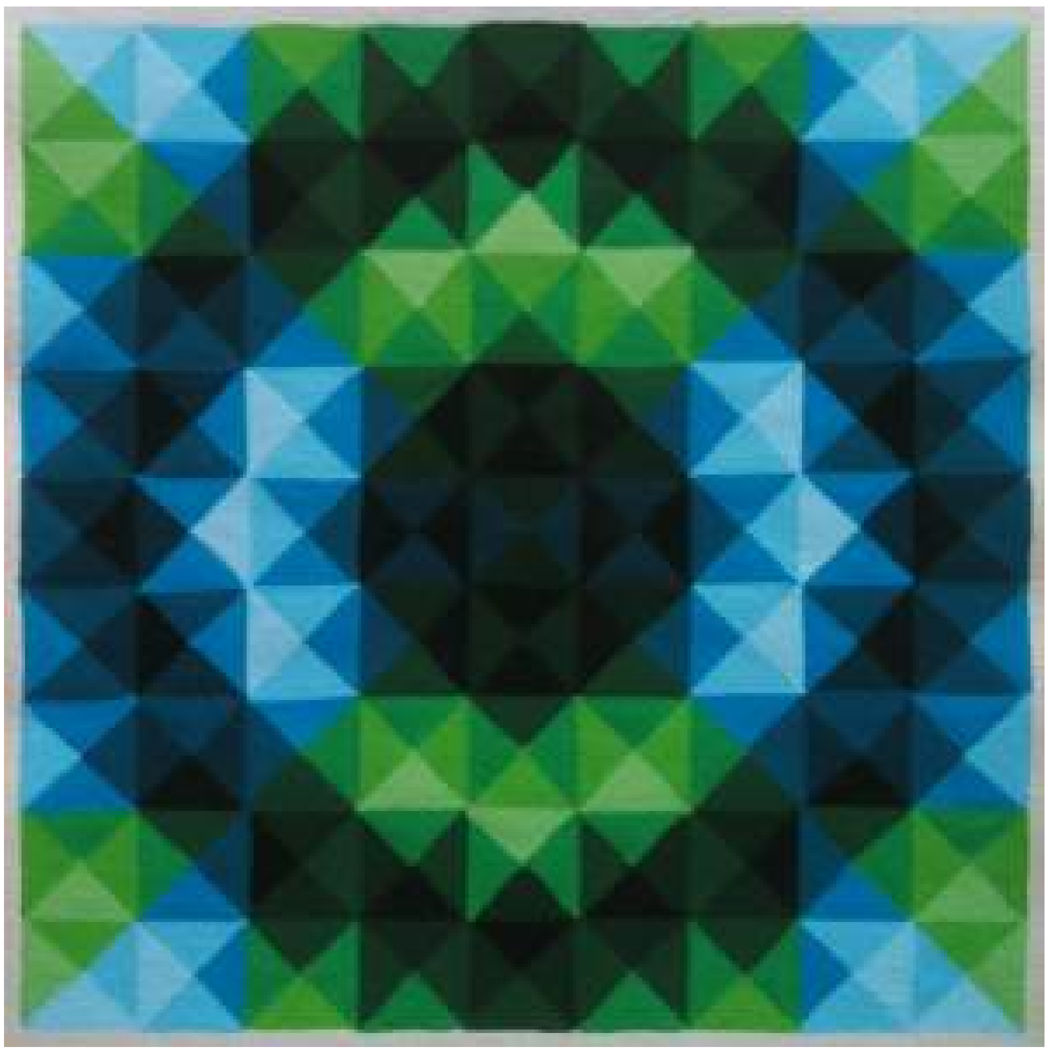
Robert Smithson, *Entropy And The New Monuments*, from "Unpublished Writings" in Robert Smithson: "The Collected Writings", edited by Jack Flam, University of California Press, 1996.

The Park Place Gallery Group



Tamara Melcher, "Untitled", 1965

The Park Place Gallery Group

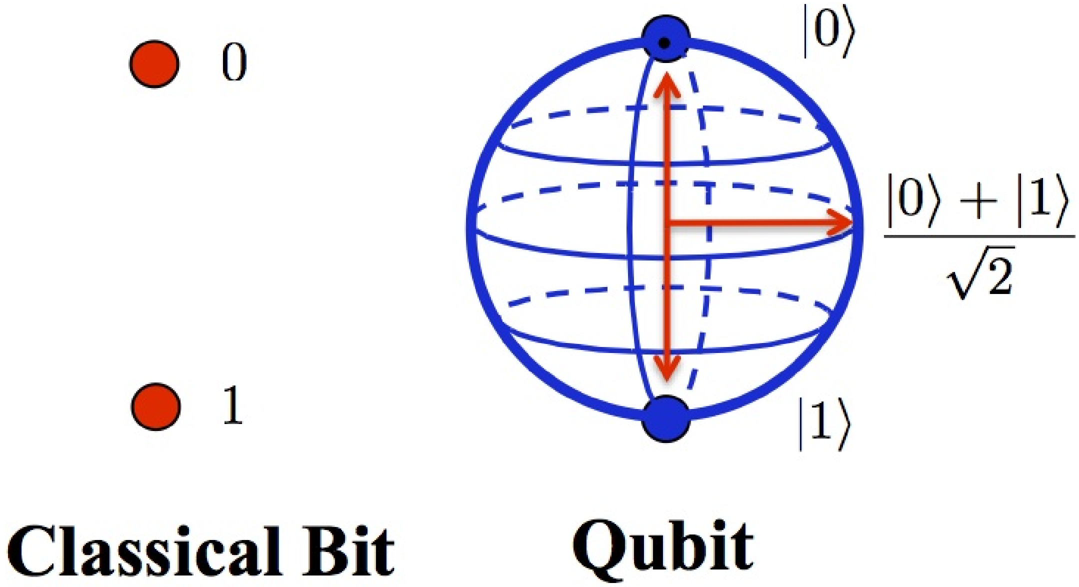


Dean Fleming, "Tunis", 1964

Quantum Information

• with Shannon entropy: fair coin 0/1, after tossing N times possibilities 2^N and $S = \log_2(2^N) = N$ number of *bits* needed to describe output

• from classical 0/1 *bits* to quantum *qbits*

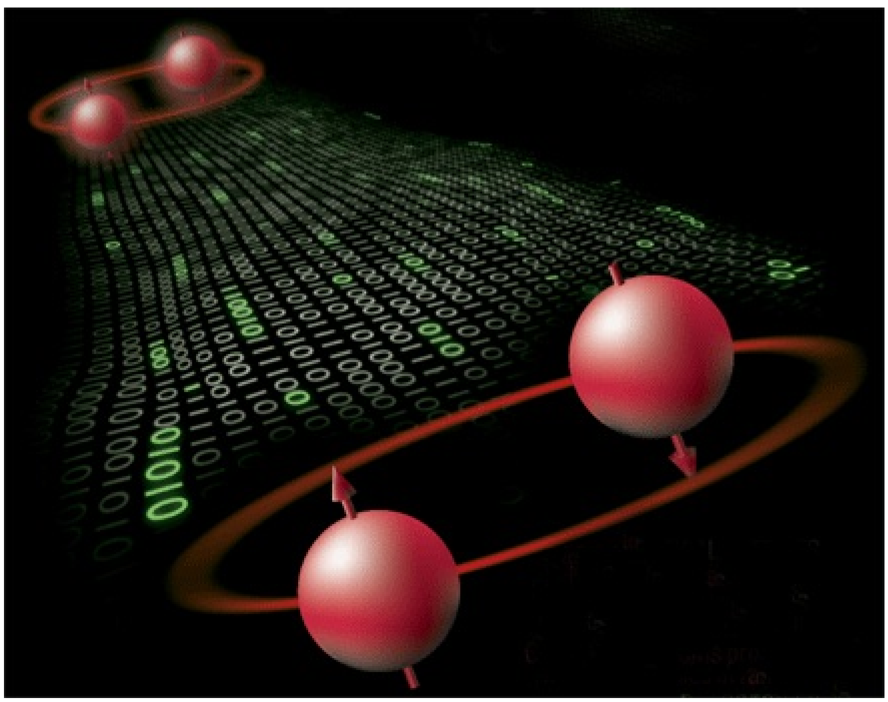


• *measurement* collapses a qbit to a bit (outcome of measurement on a qbit is a classical bit, with probabiliites)

• von Neumann entropy $S = -\mathrm{Tr}(\rho \log \rho)$

Entanglement

• the quantum state for a system of two particles does not always separate out into a contribution for each: it can be *entangled*

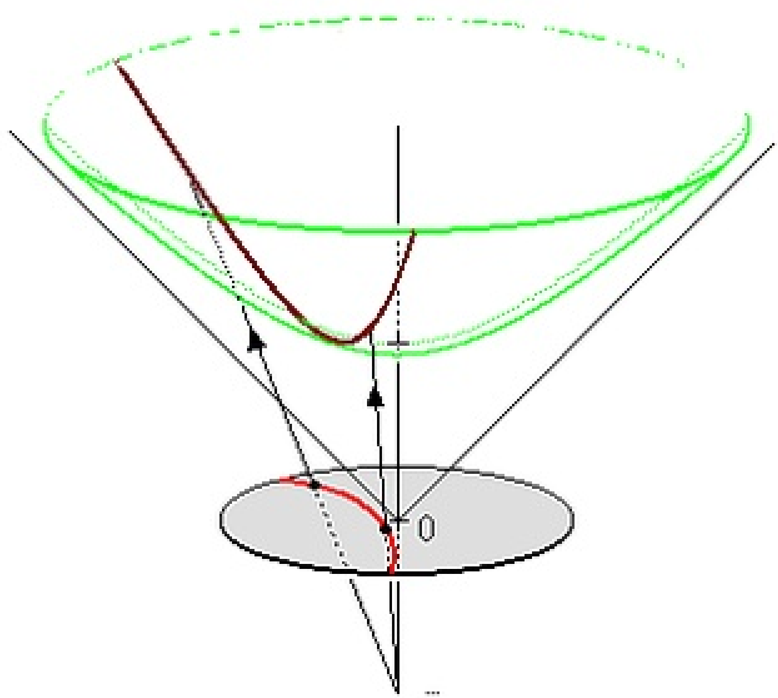


• Entanglement at the basis of the theory of quantum communication

• Entanglement can be measured using von Neumann Entropy

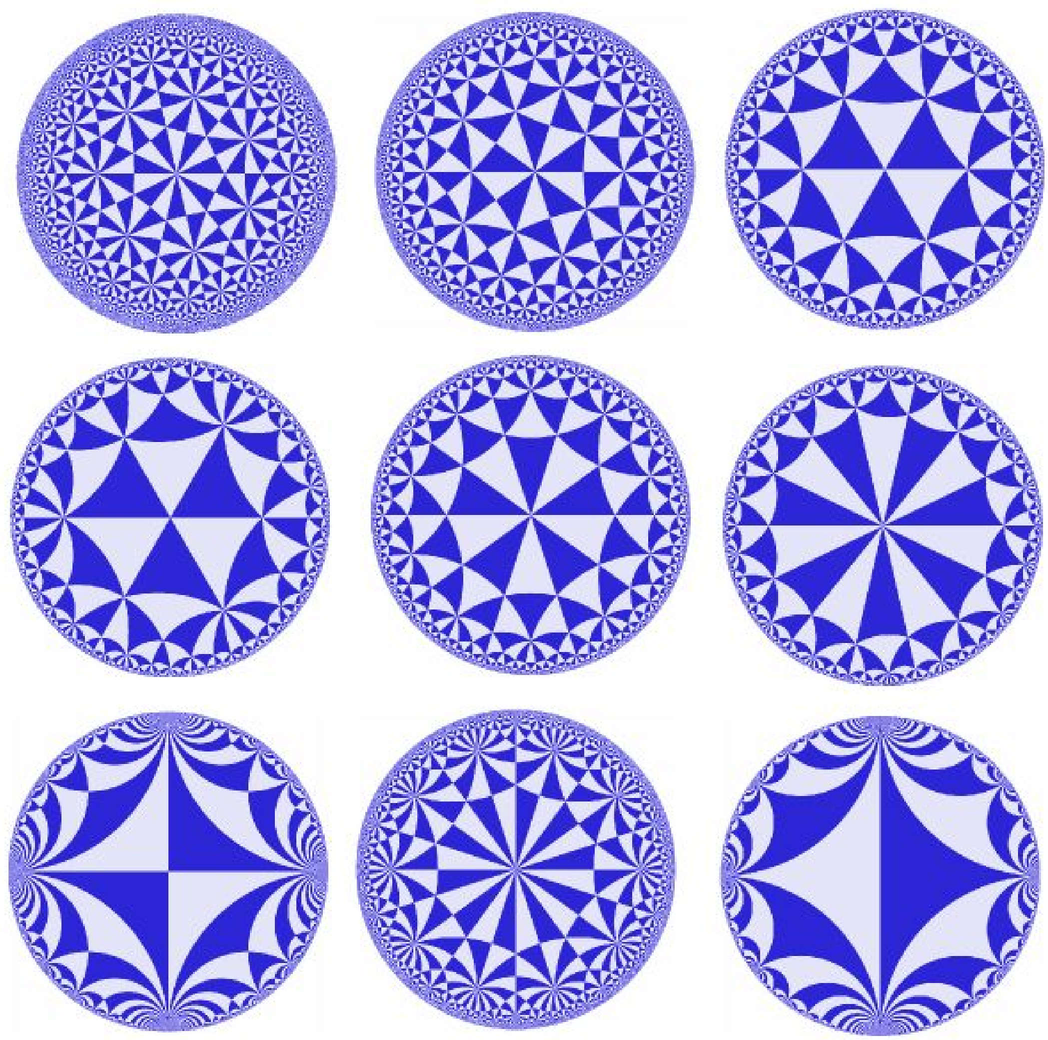
Holography Principle

• String Theory conjectures an equivalence between Conformal Field Theory on *boundary* and Gravity on *bulk* space in negatively curved AdS spacetimes

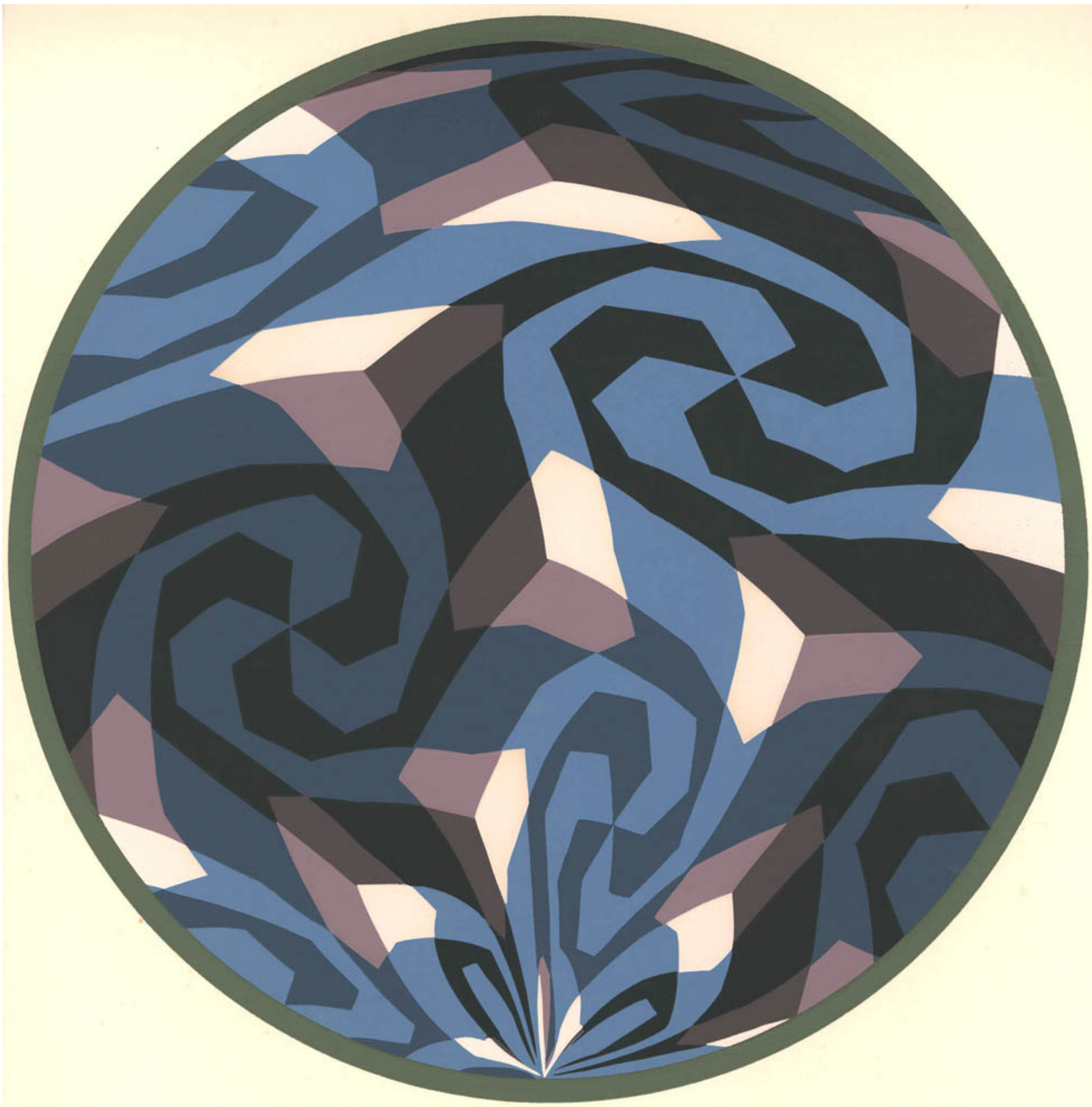


• the Hyperbolic Plane is the simplest example of such spaces (hyperboloid and disk model)

• Tilings of the hyperbolic plane: hyperbolic metric and boundary at infinity



M.C. Escher, "Circle Limit III", 1959

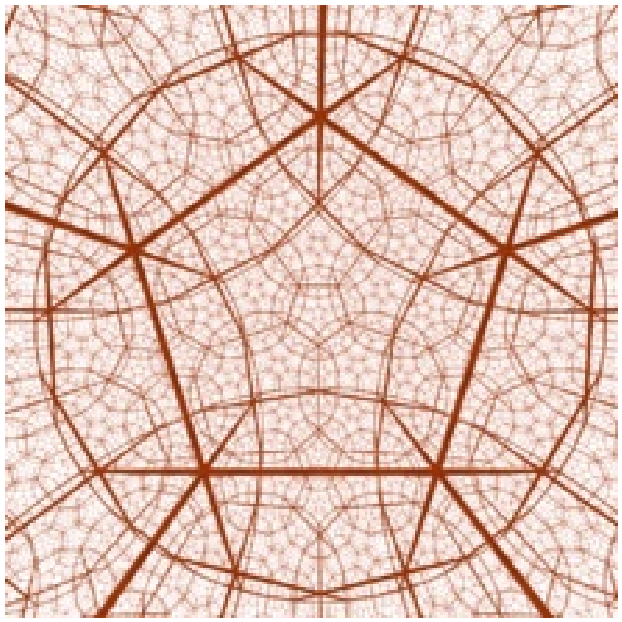


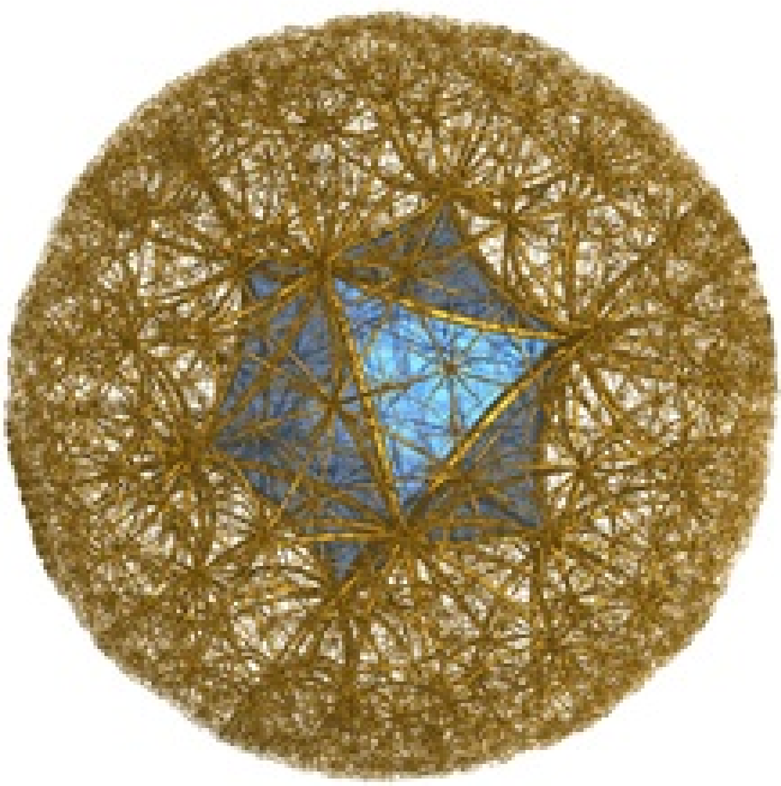
Hans Hinterretier, “Untitled”, 1967

Hans Hinterretier, "Kreis Komposition", 1978

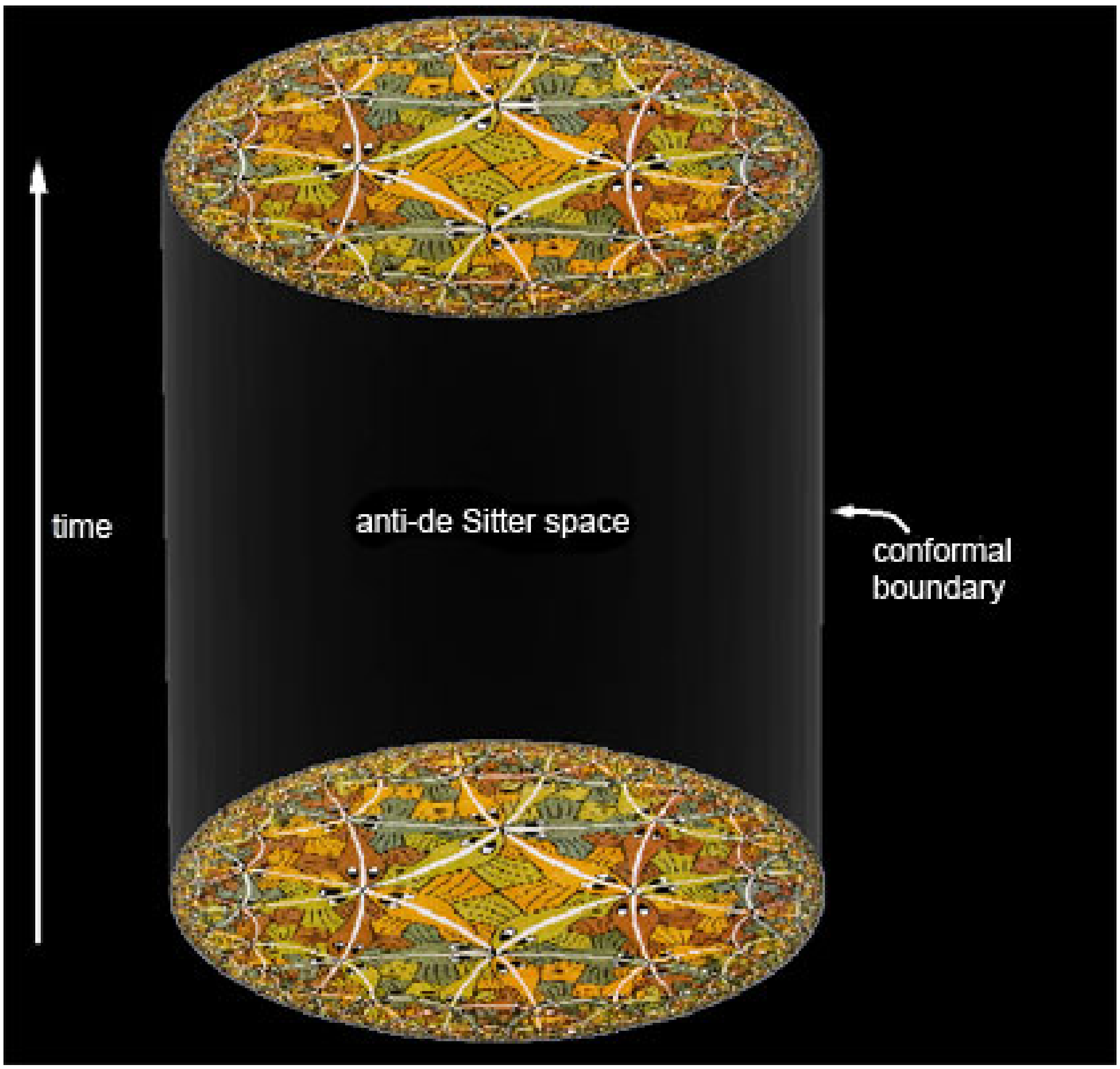
Hans Hinterretier, "Opus 67 B", 1975–1979

- Hyperbolic Space in 3D and Tilings



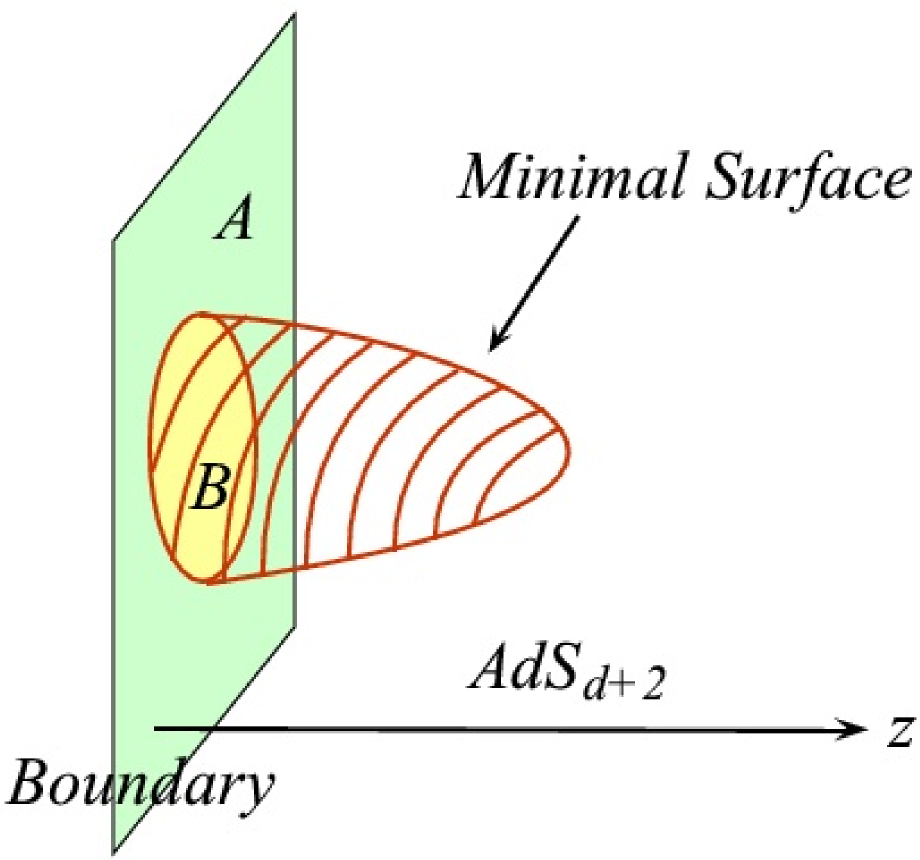


• AdS Spacetime:



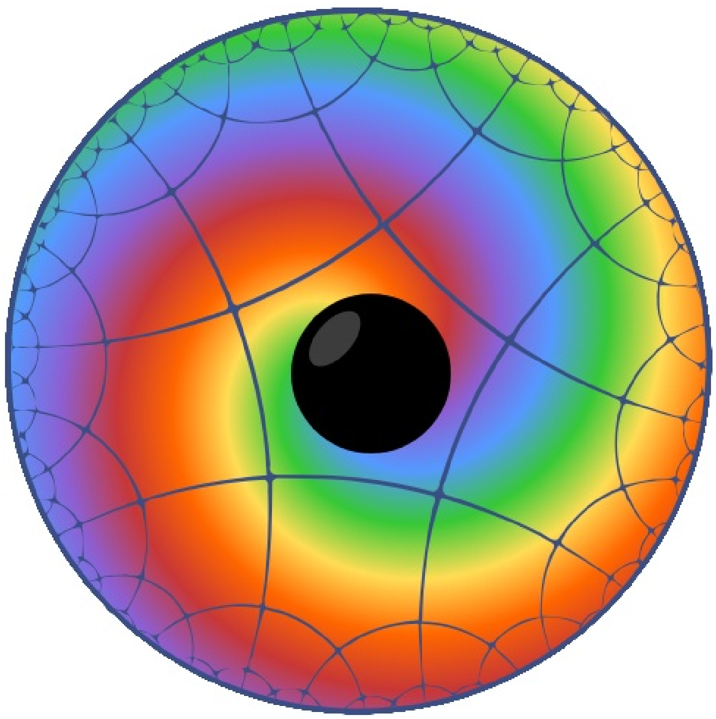
Entanglement Entropy and Holography

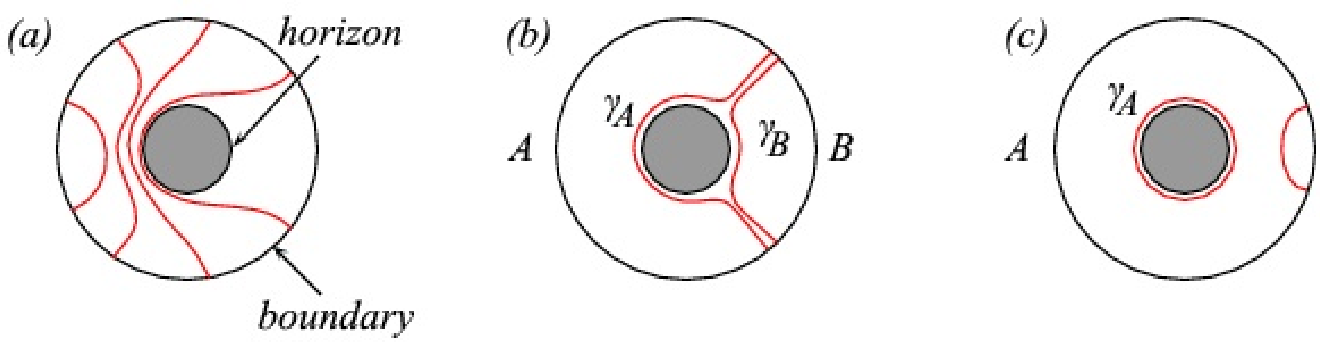
• Ryu–Takayanagi conjecture: Entanglement Entropy of quantum states on the conformal boundary = Area of minimal surface in the bulk with same boundary



• Can reconstruct gravity (Einstein equations) from properties of Entropy

• more complicated case when there is a black hole in the bulk space

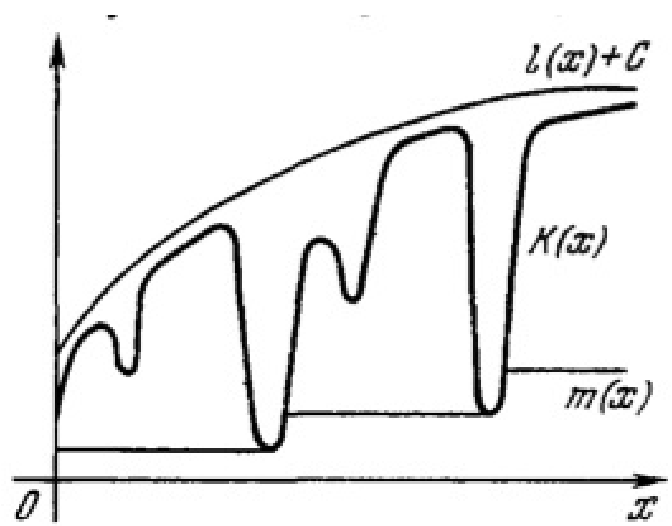




Entropy versus Complexity

• two related but *different* concepts

• **Kolmogorov complexity** of something is *shortest length of an algorithmic description* of that object (compression algorithms approximate)



• a number like 10^{100} has very low complexity compared to a random 100-digits numbers

• ... but *non-computable function* (cannot predict where dips in complexity occur: related to unsolvability of the halting problem)

• Shannon entropy as an "averaged version" of complexity

$$S(X) = \lim_{n\to\infty} \mathbf{E}(\frac{1}{n} K(X_1 \cdots X_n \,|\, n))$$

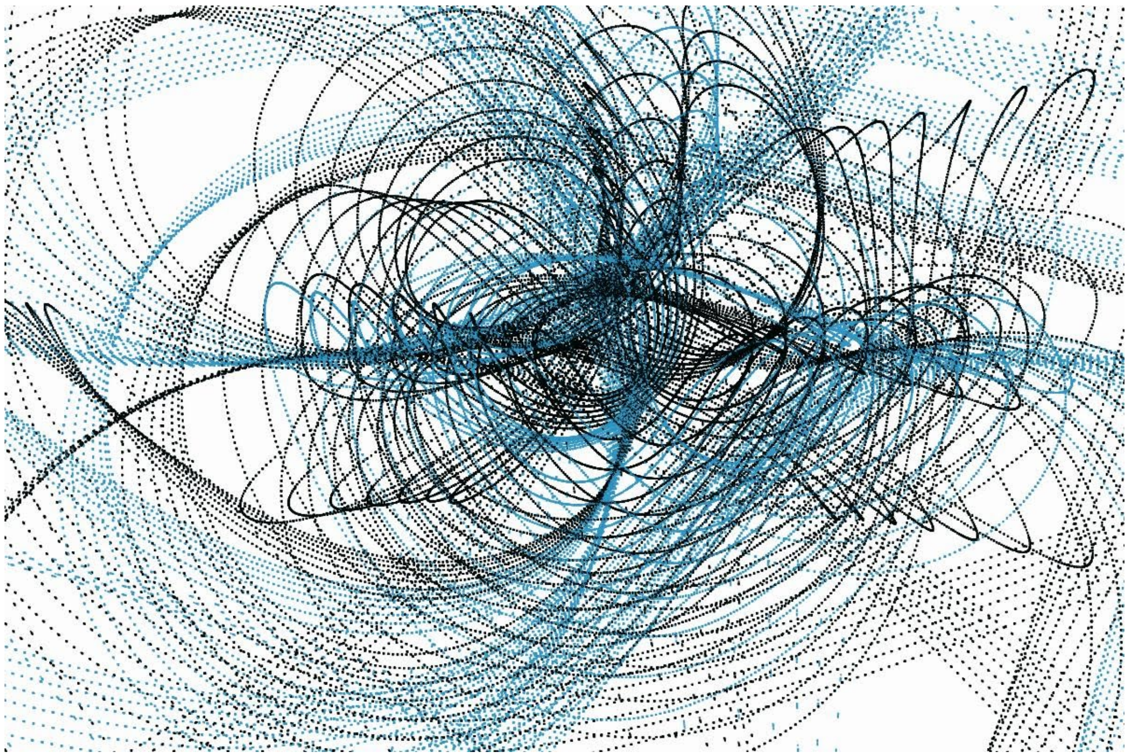
• Kolmogorov complexity is maximal for *completely random* things

• not the intuitive idea of complexity (= structures)

• Murray Gell-Mann's notion of Effective Complexity measures Kolmogorov complexity of "regular patterns"

• effective complexity is high in an intermediate region between total order and complete randomness

Gell-Mann Complexity and Generative Art



Dennis M. Callahan Jr. "Desired Constellations", 2010

- *Philip Galanter, "What is Generative Art? Complexity Theory as a Context for Art Theory"*

- `http://recoveringinfinity.blogspot.com/`

Problems with Arnheim's "Entropy and Art"

(1) *Problems with the Scientific notion of Entropy*

- *overly* emphasizes Entropy = Disorder

- confuses *Equilibrium* with *Order*

- appears to miss entirely the fact that Shannon's "Information content" is a *statistical* notion

- confuses *Entropy/Information* with *Complexity*

- misunderstands the Information Theory notion of *redundancy*

(2) *Problems with the notion of Order*

- his discussion of Order seems based on several different and not always compatible meanings

- Arnheim proposes a point of view based on Gestalt Psychology's notions of "tension reduction" and "field process" to describe Order and Structures

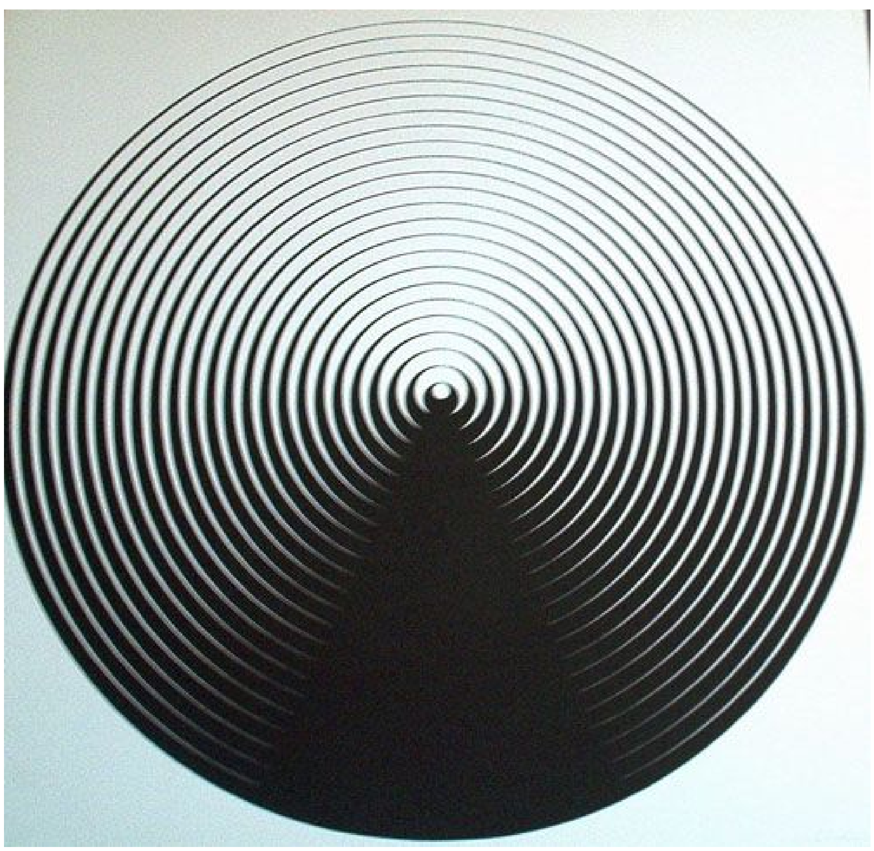
- not *predictive* notions

- Gestalt Psychology has interesting relations to mathematics of Pattern Theory, but not so much to thermodynamic entropy

- Arnheim seems unaware of where in the Art of his time Gestalt Psychology had really played a central role

Gestalt Psychology, Information, and "Kinetic and Programmed Art"

- Arte Cinetica e Programmata, developed in Italy in the 1950s, 1960s and 1970s (Bruno Munari, Gianni Colombo, Getulio Alviani, Yacoov Agam)



Getulio Alviani, "Untitled" 1964

(3) *Problems with Art interpretation*

• a misinterpretation of Andy Warhol's work: in the repetitions of images what matters are the details that are *not* identical, that's where the semantic meaning lies... the meaning is a message about the mass consumerist society

• similarly widespread misinterpretations in Arnheim's text of Minimalist Art

• interesting criticism of Arnheim's work:

- Peter Lloyd Jones, *Some Thoughts on Rudolf Arnheim's Book "Entropy and Art"*, Leonardo, Vol. 6, No. 1 (1973), pp. 29–35