Atoms and the Bohr Model

Reading: Gray: (1-1) to (1-7)
OGN: (15.1) and (15.4)
Outline of First Lecture

I. General information about the atom

II. How the theory of the atomic structure evolved
   
   A. Charge and Mass of the atomic particles
      1. Faraday
      2. Thomson
      3. Millikan
   
   B. Rutherford’s Model of the atom

Reading: Gray: (1-1) to (1-7)
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Atoms consist of:

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<tr>
<th>Mass (a.m.u.)</th>
<th>Charge (eu)</th>
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<tr>
<td>Protons</td>
<td>~1</td>
</tr>
<tr>
<td>Neutrons</td>
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<tr>
<td>Electrons</td>
<td>1/1836</td>
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| 6 C |
| 12.01 |
1 Å = 10^{-10} \text{ m}

Note: Nucleus not drawn to scale!
HOW DO WE KNOW:

- Atomic Size?
- Charge and Mass of an Electron?
- Charge and Mass of a Proton?
- Mass Distribution in an Atom?

Reading: Gray: (1-1) to (1-7)
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# A Timeline of the Atom

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Calculating the Number of Atoms in One Cubic Centimeter of Gold

Atomic weight of gold = 197.0 g / mole

\[
\frac{19.28 \text{ g}}{197.0 \text{ g / mole}} = 0.09786 \text{ moles}
\]

How many gold atoms?

\[ N_A = 6.022 \times 10^{23} \text{ atoms / mole} \]

\[
(0.09786 \text{ moles}) \times (6.022 \times 10^{23} \text{ atoms / mole}) = 5.893 \times 10^{22} \text{ atoms}
\]

There are approx. 5.9 x10^{22} atoms in 1 cm³ of gold.
Calculating Atomic Size

Assuming each atom takes up a volume of \( \frac{4}{3} \pi r^3 \), we can calculate the radius of a single atom:

\[
\frac{1.0 \text{ cm}^3}{(5.9 \times 10^{22})} = \frac{4}{3} \pi r^3
\]

\( \text{radius of one gold atom} = 1.6 \times 10^{-8} \text{ cm}, \text{ or } 1.6 \text{ angstroms} \)
The Person Behind The Science

Michael Faraday
1791-1867

Highlights

- Bookbinder turned self-taught scientist
- Discovered magnetic optical rotation
- Invented the *Dynamo*, a device capable of converting electricity into motion (1821)

Moments in a Life

- Began experimenting on electricity in 1813 under Sir Humphrey Davy
- Discovered electromagnetic induction (1831)
- Published a three volume treatise titled *Experimental Researches in Electricity* (1839-1855)
# A Timeline of the Atom

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Faraday (1830): discovered this phenomenon and determined the charge to mass ratio of ions.

Charge to mass ratio of ions: current which passes through circuit divided by mass gained on electrode.

Charge per Unit Mass of Ions
What Faraday Found

- Charge to mass ratio for various ions.
- e.g. $e/m$ for $H^+ = 10^8$ C/kg

Reading: Gray: (1-1) to (1-7)
OGN: (15.1) and (15.4)
Electroplating
Sir J.J. Thomson
1856-1940

Highlights

- Cavendish Professor of Experimental Philosophy at Cambridge University (most important position in physics at the time)
- Received his degree from Trinity College in mathematics (1880)

Moments in a Life

- Appointed Cavendish Professor (1884)
- Won the Nobel Prize in Physics (1906) for his work on the properties of the electron
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Thomson: determined the charge-to-mass ratio of electrons by balancing the force laws: $\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B}, \quad \vec{F}_{\text{elec}} = q\vec{E}$
What Thomson Found (1897)

• Found charge/mass ratio for electron: \( \sim 1.2 \times 10^{11} \text{ C/kg} \)

• \( e/m \) for electron = \( 1.2 \times 10^{11} \text{ C/kg} \), \( e/m \) for proton = \( 10^8 \text{ C/kg} \).

• Since \( 1.2 \times 10^{11} \text{ C/kg} \gg 10^8 \text{ C/kg} \), either the electron has a far greater charge than the proton, or it has far less mass.

Reading: Gray: (1-1) to (1-7)
OGN: (15.1) and (15.4)
The Person Behind The Science

Robert Millikan
1868-1953

Highlights

- Appointed Director of the Norman Bridge Laboratory of Physics, Caltech (1921)
- Received Nobel Prize in Physics (1923)
- Worked on experimental aspects of photoelectric effect

Moments in a Life

- Became a professor at the University of Chicago (1910)
- Performs his famous oil drop experiments (1909)
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Millikan: determined that charges occur in multiples of $1.60 \times 10^{-19}$ C (the charge of an electron) by balancing gravitational and electrical forces and using the previously-obtained charge to mass ratio of electrons.
## Conclusions

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<td>Atomic radius $\approx 10^{-8}$ cm</td>
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<td>$(e/m)$ for proton $\approx 10^8$ C/kg</td>
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<tr>
<td>$(e/m)$ for electron $\approx 1.2 \times 10^{11}$ C/kg</td>
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<td>$\frac{e^- \text{ charge}}{e/m \text{ for } e^-} = \frac{1.6 \times 10^{-19} \text{ C}}{1.2 \times 10^{11} \text{ C/kg}} \approx 1.3 \times 10^{-30}$ kg</td>
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<tr>
<td>(The currently accepted value is: $9.1 \times 10^{-31}$ kg)</td>
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| **Mass of the proton:** |
| $\frac{p^+ \text{ charge}}{e/m \text{ for } p^+} = \frac{1.6 \times 10^{-19} \text{ C}}{1 \times 10^8 \text{ C/kg}} \approx 1.2 \times 10^{-27}$ kg |
| (The currently accepted value is: $1.7 \times 10^{-27}$ kg) |
The Person Behind The Science

Ernest Rutherford
1871-1937

Highlights

● Worked for J.J. Thomson (1895)

● The scientific father of 10 Nobel Prize Laureates

● Worked on alpha, beta, and gamma particles

Moments in a Life

● Performs particle deflection experiments (1907)

● Awarded Nobel Prize in Chemistry (1908)

● Assumes position at Cambridge formerly held by J.J. Thomson (1919)
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Mass Distribution in an Atom

Rutherford: showed that the mass of an atom is not distributed evenly (otherwise every angle of scattering would be close to zero)
Rutherford thus showed that the nucleus is very dense and that the remainder is virtually all vacuum.

Rutherford’s Model of the Atom (1911)
Rutherford Backscattering Spectroscopy
**Useful Definitions:**

**Element:** A substance containing atoms of only one type, i.e. atoms with the same number of protons.

**Isotopes:** Atoms of the same element having different masses—having different numbers of neutrons.

**Ions:** Positively or negatively charged atoms—have different numbers of electrons.

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<th>Elements</th>
<th>Isotopes</th>
<th>Ions</th>
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<tr>
<td>He, Helium</td>
<td>H, Hydrogen</td>
<td>H^+, hydrogen ion</td>
</tr>
<tr>
<td></td>
<td>²H, deuterium</td>
<td>H^−, hydrogen ion</td>
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Atoms Consist of Protons, Neutrons, Electrons

- **Protons:** 1 a.m.u. +1 charge
- **Neutrons:** 1 a.m.u. 0 charge
- **Electrons:** 1/1836 a.m.u. -1 charge

- **Atomic Radius:** ≈1 Å (= 10^{-10} m)
- **Most of the Mass is in the Nucleus:** 10^{-5} Å

Reading: Gray: (1-1) to (1-7)
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How do we know which element is which?

Reading: Gray: (1-1) to (1-7)
OGN: (15.1) and (15.4)
Emission Spectra

*Each element has characteristic emission lines*
Properties of Light

General relationship: \( c = \nu \lambda \)
The Person Behind The Science

Max Planck
1858-1947

Highlights

- Early worked focused on thermodynamics
- Received Nobel Prize in Physics (1918)
- Father of the quantum revolution

Moments in a Life

- Received his doctorate from Universities of Munich and Berlin under the guidance of Kirchhoff and Helmholtz (1879)
- Published his work on quanta (1900)
Other Useful Relations:

\[ E = h \nu \]

\[ E = \frac{hc}{\lambda} \]

\[ \bar{\nu} = \frac{1}{\lambda} \]

\[ E = hc\bar{\nu} \]

*E*: energy  
*h*: Planck’s constant  
*\nu*: frequency  
*\bar{\nu}*: wavenumber  

\[ \nu = \frac{c}{\lambda} \]
Emission Spectra

*Each element has characteristic emission lines

\[ E = h\nu \] gives \( E \) for each line
Lyman Lines:

Examples:

- $E = 13.6 \text{ eV} \left( 1 - \frac{1}{n^2} \right)$, $n = 2$
- $E = 13.6 \text{ eV} \left( 1 - \frac{1}{n^2} \right)$, $n = 3$

General Case:

$E = 13.6 \text{ eV} \left( 1 - \frac{1}{n^2} \right)$, $n \in \mathbb{Z} > 1$

shorter wavelength, higher energy
Johann Balmer's doctorate from Basel was for a dissertation on the cycloid. He taught in Basel all his life both as a school teacher and as a university lecturer at the University of Basel. His main field of interest was geometry.

However Balmer is best remembered for his work on spectral series and his formula, given in 1885, for the wavelengths of the spectral lines of the hydrogen atom. The reason why the formula holds was not understood until the work of Niels Bohr in 1913.
Balmer Lines:

Examples:

\[ E = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3 \]

\[ E = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 4 \]

General Case:

\[ E = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n \in \mathbb{Z} > 2 \]

shorter wavelength, higher energy
Paschen Lines:

Examples:

\[ E = 13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n = 4 \]

\[ E = 13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n = 5 \]

General Case:

\[ E = 13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n \in \mathbb{Z} > 3 \]
Combined Formula for E:

shorter wavelength, higher energy

Lyman lines (U.V.) Balmer lines (Visible) Paschen lines (Near Infrared)

\[ E = 13.6\text{eV} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad n_1, n_2 \in \mathbb{Z} | n_2 > n_1 \]

With available values, the Rydberg constant was determined

\[ E = \hbar c R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

\[ R_H = \frac{e^2}{2\hbar c \ 4\pi\epsilon_0 a_o} = 109,677.581 \pm 0.007 \text{cm}^{-1} \]
Forms of the Rydberg constant:

\[ E = R_H \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \]

\[ R_H = \frac{\hbar c}{13.6 \text{eV}} = 9.118 \times 10^{-6} \text{cm} \] (wavelength)

\[ R_H = \frac{13.6 \text{eV}}{h} = 3.288 \times 10^{15} \text{Hz} \] (frequency)

\[ R_H = \frac{13.6 \text{eV}}{hc} = 1.097 \times 10^5 \text{cm}^{-1} \] (wavenumber)
The Classical Paradox of Atoms

Potential Energy \( \equiv PE = \frac{(-e)(+e)}{4\pi\varepsilon_0 r} = \frac{-e^2}{4\pi\varepsilon_0 r} \)

Kinetic Energy \( \equiv KE = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_e v_e^2 \)

Total Energy = PE + KE

But, lowest energy, i.e. ground state, is when?

PE \( \rightarrow 0 \) as \( r \rightarrow \infty \), but PE \( \rightarrow -\infty \) as \( r \rightarrow 0 \)

KE = 0 at \( v_p=0 \) and \( v_e=0 \)

So, lowest E is -\( \infty \) when electron is at nucleus!

Alternatively, why doesn’t atom radiate EM waves?
The Person Behind The Science

Niels Bohr
1885-1962

Highlights

• Worked with J.J. Thomson (1911)

• Awarded Nobel Prize in Physics (1922)

Moments in a Life

• Received his doctorate degree from Copenhagen University (1911)

• Professor of Theoretical Physics at Copenhagen University (1916)
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Bohr’s Theory

1. Atoms have well-defined electron orbits.
2. They don’t radiate.
3. Circular orbits: only specific orbits with specific angular momenta, \( l = n(\ ) \), are allowed \{quantization postulate\}
4. Transitions in energy: electrons go from one orbit to the next

\[
l = n \frac{\hbar}{2\pi}
\]

\( n \in \mathbb{Z} \)
Balance of forces within an atom

\[ F_{\text{coulombic}} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2} = \frac{mv^2}{r_n} \]

\((Z = \text{nuclear charge})\)

So, if we know \(v\), we know \(r_n\)
Angular momentum: solve for $v$

$$l = m \, v \, r_n = \frac{n \, h}{2 \, \pi}$$

$$v = \frac{n \, h}{2 \, \pi \, m \, r_n}$$

since we also have:

$$\frac{Ze^2}{4\pi\varepsilon_0 r_n^2} = \frac{m \, v^2}{r_n}$$

we can substitute for $v$ and solve for $r_n$:

$$\frac{Ze^2}{4\pi\varepsilon_0 r_n^2} = \frac{mn^2h^2}{4\pi^2m^2r_n^3} \rightarrow r_n = \frac{n^2h^2\varepsilon_0}{\pi m Ze^2} = \frac{n^2a_0}{Z}$$

where $a_o$ is the Bohr radius:

$$a_o = \frac{h^2 \varepsilon_o}{\pi \, m \, e^2} = 0.529 \, \text{Å}$$
Radii of Hydrogen Atom Orbitals

\[ r_n = n^2 a_0 \]

\((Z=1)\)

- For \( n = 1 \), \( r = 1^2 (0.529) = 0.529 \text{ Å} \)
- For \( n = 2 \), \( r = 2^2 (0.529) = 2.116 \text{ Å} \)
- For \( n = 3 \), \( r = 3^2 (0.529) = 4.761 \text{ Å} \)
Orbital Energies:

E = kinetic energy + potential energy

\[ E = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 r_n} \]

\[ = \left(\frac{e^2}{2r_n} - \frac{e^2}{r_n}\right)\left(\frac{Z}{4\pi\varepsilon_0}\right) \]

\[ = -\frac{e^2}{2r_n}\left(\frac{Z}{4\pi\varepsilon_0}\right) \]

\[ = \frac{-Z^2me^4}{8n^2h^2\varepsilon_0^2} \]

\[ = \frac{Z^2 \times \text{constant}}{n^2} \quad \text{constant} = -13.6 \text{ eV} \]

\[ Z = 1 \quad \text{for hydrogen} \]
Energies of Hydrogen Atom Orbitals

\[ E_n = \frac{-13.6}{n^2} \text{ [eV]} \]

\[ r_n = n^2a_0 \]

\((Z=1)\)

\[ n = 1, \ r = 1^2(0.529) = 0.529 \text{ Å} \]

\[ E = \frac{-13.6}{n^2} = -13.6 \text{ eV} \]

\[ n = 2, \ r = 2^2(0.529) = 2.116 \text{ Å} \]

\[ E = \frac{-13.6}{n^2} = -3.4 \text{ eV} \]

\[ n = 3, \ r = 3^2(0.529) = 4.761 \text{ Å} \]

\[ E = \frac{-13.6}{n^2} = -1.5 \text{ eV} \]
Radii and Energies with Differing Nuclear Charge

\[ r = \frac{n^2a_0}{Z}, \quad E = \frac{-13.6 Z^2}{n^2} \]

<table>
<thead>
<tr>
<th>Element</th>
<th>n</th>
<th>Z</th>
<th>( r )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1</td>
<td>1</td>
<td>( \frac{1^2a_0}{1} )</td>
<td>(-13.6 (1^2))</td>
</tr>
<tr>
<td>He(^+)</td>
<td>1</td>
<td>2</td>
<td>( \frac{1^2a_0}{2} )</td>
<td>(-13.6 (2^2))</td>
</tr>
<tr>
<td>Li(^{2+})</td>
<td>1</td>
<td>3</td>
<td>( \frac{1^2a_0}{3} )</td>
<td>(-13.6 (3^2))</td>
</tr>
</tbody>
</table>
The Rydberg Constant:

As measured: \( R_H = 109\,677.6\, \text{cm}^{-1} \) (correct value)

As predicted: \( R_H = 109\,737.8\, \text{cm}^{-1} \)

In order to obtain a more accurate predicted value for \( R_H \) the reduced mass must be used:

\[
m_{\text{reduced}} = \frac{m_e m_p}{m_e + m_p} \quad (\text{to account for the fact that a proton has mass also})
\]
Success of the Theory

- Also Gets All Other 1e⁻ Atoms Right!
- So Bohr Gets the H atom Right!
- Go to Stockholm, Niels!
Failures of the Theory

- So Bohr Gets the H atom Right!
- Also Gets All Other 1e⁻ Atoms Right!
- Go to Stockholm, Niels!

**BUT**

There are two major flaws:
(a) Can’t explain any multi-electron atoms!
(b) Is completely wrong, according to quantum mechanics!
Atoms and the Bohr Model

End

Reading: Gray: (1-1) to (1-7)
OGN: (15.1) and (15.4)