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Discussion of paper by F. Miao and M. Ghosn "Modified subset simulation method for reliability analysis of structural systems", Structural Safety, 33:251–260, 2011

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The subject paper presents a 'Regenerative Adaptive Subset Simulation' (RASS) algorithm that includes some modifications to the original Subset Simulation algorithm for calculating small failure probabilities for dynamic systems that was first proposed by Au and Beck in [1]. In particular, the authors state that their proposed modifications overcome some limitations of the original Metropolis–Hastings algorithm used in Subset Simulation, including the 'burn-in' problem and the difficulty of the selection of the proposal distribution. This discussion intends to clarify several issues associated with the paper.

(1) The most important point that we wish to make is that SubSim (Subset Simulation) does not suffer from 'burn-in', despite the fact that this is a common feature of applications of the MH (Metropolis–Hastings) algorithm, because of the way that SubSim works. This point was noted when SubSim was introduced [1] and in subsequent developments. In the subject paper, it is mentioned at several points (e.g., abstract; last but one paragraph in Introduction; point 2 of Summary Section 2.4, and at the start of Section 4.1) that the MH algorithm used in SubSim has a 'burn-in' problem, meaning that it takes a number of initial samples (generally unknown) for the distribution of the Markov chain samples to converge to the stationary target distribution, which is an intermediate conditional distribution in SubSim. While this 'burn-in' issue is generally true for the MH algorithm, it is not true for SubSim because for a given intermediate failure level, the MH algorithm is seeded with conditional samples selected from the previous failure level that lie in the current failure level (guaranteed by its construction); that is, the samples that seed the multiple Markov chains in a given level are always distributed according to their target distribution, being the original probability distribution of the input variables conditional on the intermediate failure event. This means that all of the Markov chains generated in SubSim are in the stationary state right from the start. This is called *perfect sampling* in the MCMC literature and it is rare for an MCMC algorithm to exhibit it.

(2) As the 'burn-in problem' is irrelevant to SubSim, strategies on 'regeneration' as presented in Section 4.1 and 4.2 seem no longer relevant. They may create unnecessary complication to the algorithm and make it computationally more expensive.

(3) It is mentioned in the paper that the original Metropolis–Hastings (MH) algorithm was used in the original SubSim method [1] but, in fact, a Modified Metropolis–Hastings (MMH) algorithm was proposed because the original MH algorithm is very inefficient in high dimensions; the reason is that the acceptance probability in the MH accept/reject step becomes very small as the dimension increases so that large numbers of repeated samples are obtained, slowing down the sample coverage of the important regions of the intermediate failure domains. The MMH algorithm proposed in [1] overcomes this deficiency by performing component-wise generation and acceptance/rejection of candidate samples, rather than at the vector level of the original MH algorithm; that is, instead of using a high-dimensional proposal distribution to directly obtain the candidate vector, in the MMH algorithm a sequence of one-dimensional proposals is used to generate the candidate vector one component at a time.

(4) In several places (e.g. point 4 of Summary Section 2.4), it is mentioned that it is hard to choose the spread of the proposal distribution for the MH algorithm. In [1] it was recommended for SubSim that this spread be taken equal to the standard deviation of the input variables. Applications to different engineering problems (e.g., [2–4]) suggest that this choice does give a robust procedure. In fact, the choice of the proposal distribution has been automated in a recent implementation of SubSim under a spreadsheet environment [5], where the only algorithm-related parameters to choose are the number of samples at each simulation level and the conditional failure probability which controls the adaptive choice of successive intermediate failure events. Of course, the spread of the proposal distribution can always be optimized for a particular type of problem at some computational cost, although the gain in efficiency may not be substantial. For example, a recent study [6] shows that the optimal choice does not lead to a substantial reduction in the coefficient of variation of the failure probability estimator for a given number of samples when compared with the recommendation made in [1].

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(5) For the original SubSim algorithm, the generation of an additional Markov chain sample requires one evaluation of the response function (e.g., one structural analysis). According to the proposed RASS algorithm, more than one evaluation of the response function is required in generating one additional Markov chain sample, in order to implement the delayed-rejection. The actual computational effort in this case should thus be directly measured by the number of response function evaluations rather than the number of Markov chain samples generated, with the former being reported to allow comparison. It is not clear whether the number of samples reported for RASS in the tables correspond to the number of function evaluations or the number of Markov chain samples.

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