Investors' Behavior on S&P 500 Index during Periods of Market Crashes: A Visibility Graph Approach

Michail D. Vamvakaris[1], Athanasios A. Pantelous[1*], Konstantin Zuev[2]

[1]Department of Mathematical Sciences and Institute for Risk and Uncertainty, University of Liverpool, Peach Street L69 7ZL, Liverpool United Kingdom

[2]Department of Computing and Mathematical Sciences, California Institute of Technology, E. California Blvd. Mail Code 305-16, Pasadena, CA 91125 USA

Abstract

Investors' behavior in the market is highly related to the properties that financial time series capture. Particularly, nowadays the availability of high frequency datasets provides a reliable source for the better understanding of investors' psychology. The main aim of this chapter is to identify changes in the persistency as well as in the local degree of irreversibility of S&P 500 price-index time series. Thus, by considering the US stock market from 1996 to 2010, we investigate how the Dot.com as well as the Subprime crashes affected investors' behavior. Our results provide evidences that Efficient Market Hypothesis does not hold as the high frequency S&P 500 data can be better modeled by using a fractional Brownian motion. In addition, we report that both crises only temporary effect investors' behavior, and interestingly, before the occurrence of these two major events, the index series exhibited a kind of "nervousness" on behalf of the investors.

JEL classification: C02; C55; G02; G14

Keywords: High Frequency Data; S&P 500, Hurst Exponent, Irreversibility, Visibility Graph Method

*Address correspondence to Athanasios A. Pantelous, Department of Mathematical Sciences and Institute for Risk and Uncertainty, University of Liverpool, Peach Street, L69 7ZL Liverpool, United Kingdom, e-mail: A.Pantelous@liverpool.ac.uk, Tel: +44 (0)151 794 5079.

Investors' Behavior on S&P 500 Index during Periods of Market Crashes: A Visibility Graph Approach

Version: 13th November 2016

Abstract

Investors' behavior in the market is highly related to the properties that financial time series capture. Particularly, nowadays the availability of high frequency datasets provides a reliable source for the better understanding of investors' psychology. The main aim of this chapter is to identify changes in the persistency as well as in the local degree of irreversibility of S&P 500 price-index time series. Thus, by considering the US stock market from 1996 to 2010, we investigate how the Dot.com as well as the Subprime crashes affected investors' behavior. Our results provide evidences that Efficient Market Hypothesis does not hold as the high frequency S&P 500 data can be better modeled by using a fractional Brownian motion. In addition, we report that both crises only temporary effect investors' behavior, and interestingly, before the occurrence of these two major events, the index series exhibited a kind of "nervousness" on behalf of the investors.

JEL classification: C02; C55; G02; G14

Keywords: High Frequency Data; S&P 500, Hurst Exponent, Irreversibility, Visibility Graph Method

1. Introduction

The *Efficient Market Hypothesis* (EMH) is one of the milestones in the modern financial theory. It was developed independently by Samuelson (1965) and Fama (1963, 1965), and in a short time, it became a guiding light not only to practitioners, but also to academics. In brief, EMH states that in an efficient market, stocks incorporate instantly all publicly available information useful in evaluating their prices and, thus no one can consistently outperform the overall market without accepting higher risk. Since information arrives randomly, the stocks' future price trajectories are as predictable as a sequence of cumulated random numbers. In other words, price increments (decrements) are *independent*, which implies that no one can predict stocks' price future paths based on their history. Although, EMH has been developed and discussed extensively since then, actually the concept that price increments are independent, uncorrelated, and unpredictable was initially introduced by Louis Bachieler. Bachelier (1900) claimed that a security price sequence can be modelled by a *stationary* Gaussian random walk and, consequently, price increments do not contain autocorrelation. This idea was developed further by Osborn (1959), who proposed that the logarithm of price changes behave similarly to particles in Brownian motion.

On the other hand, by studying cotton price series, Mandelbrot (1967, 1997) observed that the distribution of price increments was *not* Gaussian, but rather stable Paretian, which implies that the distribution is more peaked around the mean and has fatter tails, which actually means that outliers are more frequent compared to those supported by the Normal distribution frame. Furthermore, his research concluded that cotton price series are not stationary and that there is dependence among the price increments. This latter finding led him to conclude that stock price series should be best modelled by a *fractional Brownian motion* (fBm) (Mandelbrot and Van Ness, 1968), which is a long-memory version of the standard Brownian motion (sBm) process characterized by the Hurst exponent, H. Even though, Fama (1963, 1965) studied stock price series and he also exhibited the distinguishing leptokurtosis that characterizes the distribution of stock price increments, and he was adamant that price changes are independent and uncorrelated.

Since then, bulk of literature focuses mainly on studying whether the EMH is valid, i.e., whether there is a random walk in the markets, then based on the analysis of different empirical data, contradictory evidences have been demonstrated. Obviously, since EMH should not be approached as an *all-or-nothing* question (Lim, 2007), several studies have demonstrated the dynamic character of markets' efficiency (Cajueiro and Tabak, 2004a; Grech and Mazur, 2004; Lim, 2007; Alvarez-Ramirez et al., 2008; Grech and Pamuła, 2008; Lim et al. 2008; Barunik and Kristoufek, 2010; Wang et al., 2010). What is more, Campbell et al. (1997) claimed that it is better to study the relative efficiency of a market, since the hypothesis of existence of an efficient market seems to be a utopia in practice, thus it is better to distinguish between more and less efficient periods or markets.

In this chapter, we have been motivated by Grech and Mazur (2004) and Lim et al. (2008) who showed that DJIA index became more "*nervous*" before some major market crash events and that stock market crashes deteriorate temporarily the market efficiency in Asian countries, respectively. Under this frame, since the stock markets' time series are the mirror of the investors' behavior, we study how particular financial crashes, such as the Dot.com crisis in 2000 and the Subprime Mortgage crisis in 2007 affected the behavior of S&P 500 price-index using high frequency data by quantifying the degree of index series persistency, as well as quantifying the memory in the index series. It is the first time that instead of just using tests to reject or confirm the random walk hypothesis we take advantage of the recently introduced Visibility Graph method to identify changes of investors' behavior throughout periods of financial crashes by studying how the local properties of the index series evolve over the course of time.

By using high frequency data our results provide stronger evidence that stock markets have a dynamic character and thus, properties of financial time series might vary over time. In particular, we find that the impact of two specific financial crashes in the S&P 500 index series, and actually on its investors' behavior, is only temporal while we also provide evidences that the index time series exhibit "nervousness" before those events occurred. To the best of our knowledge, this is the first attempt to study the Hurst exponent of stock indexes using high frequency data. Finally, we report that there is a link between the Hurst exponent and the degree of irreversibility of a series.

The paper is structured as follows. In Section 2, we analyze the properties of the Hurst exponent and what the irreversibility in financial time series suggests. In Section 3, the proposed method is explained in order to calculate the Hurst exponent and to quantify index series irreversibility. Section 4 describes the data we use and Section 5 provides the methodology followed. Finally, in Section 6, the empirical results are presented and Section 7 concludes the whole discussion.

2. Literature Review

2.1 Hurst Exponent

In 1956, the hydrologist Harold Edwin Hurst (Hurst, 1956) investigated a novel method for calculating the long-range dependences within observations of a time series. In fact, Hurst exponent, H, estimates the autocorrelations of a series as well as the rate at which autocorrelations diminish as the time delay between pairs of values increases.

The values of the index, H, range within $[0,1]$. Values in the interval $H \in (0.5, 1]$ indicate *a positive auto-covariance coefficient at all lags* which points out a persistent behavior (Mandelbrot and Van Ness, 1968). Interpreting persistency in terms of financial time series, it implies that an increment (decrement) is, statistically speaking, more probable to be followed by another increment (decrement). According to Greene and Fielitz (1977), persistency is not the result of an ordinary serial dependence, but rather its' root can be traced on a special kind of dependence with an infinite memory which is called "*non-cyclic, long-run statistical dependence*". Time series that are characterized as persistent contain long-memory and thus, they exhibit "*trends*" and "*cycles*" of varying length.

On the other hand, $H \in [0, 0.5)$ characterizes time series which have *negative* auto-covariance at all lags and series exhibit an anti-persistent behavior (Barkoulas et al., 2000). This means that a decrement (increment) is more probable to be followed by an increment (decrement). In other words, an anti-persistent series reverts more frequently than a random one (Kristoufek, 2010). Finally, when $H = 0.5$, we deal with a process which does not contain dependences, such as a sBm process.

The EMH states that technical analysis cannot help the markets' investors to gain additional information that facilitates them to predict future movements of prices, since prices' future paths are not dependent on their history. Consequently, if a market was efficient, then its behavior could be modelled by a random walk, i.e., $H = 0.5$, which indicates that the series do not contain auto-correlation. Consequently, any deviation from $H = 0.5$ is an indication of presence of auto-correlation in the series which in turn indicate a violation of the EMH. According to Morales et al. (2012) due to the finite size of financial time series, it is also possible to obtain a Hurst exponent different than $H = 0.5$, even for random series. So, in most cases there is a small interval near the value $H = 0.5$, where we can assume that the series increments are independent and uncorrelated. Fractal Brownian series with higher values of Hurst exponent are found to be smoother in comparison with lower values of the exponent. In **Fig. 1**, we plot fBm for different values of the Hurst exponent in order to present clearly the additional information gained.

Along these lines, rich literature in finance has been developed to study the Hurst exponent and connect it with the existence of EMH (Cajueiro and Tabak, 2004a,b; Grech and Mazur, 2004; Di Matteo, 2007; Alvarez-Ramirez et al., 2008; Grech and Pamuła, 2008; Lim et al., 2008; Barunik and Kristoufek, 2010; Wang et al., 2010). In more details, Cajueiro and Tabak (2004a,b) calculated the time-varying local Hurst exponent using daily data of 11 emerging and 2 developed stock indexes from January 1992 till December 2002. Even though, the Hurst exponent for the emerging markets was found to be considerably higher than 0.5, the long term trajectory of H was decreasing eventually towards 0.5, which indicates that the efficiency of these markets was increasing over the course of time. On the other hand, the efficiency of developed markets appeared not to have serious changes over time, and to fluctuate around $H = 0.5$. The same conclusion was also confirmed by Wang et al. (2010) who studied the efficiency of Shanghai stock market. On the other hand, Di Matteo (2007) examined 32 stocks' market indexes for one time-period in order to study the multi-scaling properties of financial time series and her results illustrate that developed stock markets were characterized by H close to 0.5, while in the emerging markets the estimated Hurst exponent was much higher. Grecha and Mazurb (2004) used the concept of local Hurst exponent as a tool for forecasting a market crash. Using daily data, they analyzed three major market crashes, such as the market crashes of 1929 and 1987 in US and of 1998 in Hong Kong. They reported that the local Hurst exponent dropped sharply before the crashes, which is an indication that the market becomes more "*nervous*" and "*volatile*" before some major events. On contrary, throughout the crashes, the values of H increased significantly indicating a ramped up inefficiency. The same conclusion was also reported by Grech and Pamuła, (2008) for the Polish stocks' market. Using again daily data, Alvarez-Ramirez et al. (2008) studied the evolution of S&P 500 price-index and DJIA index as well. Their results also exhibited a time-varying evolution of market efficiency. Furthermore, Lim et al. (2008) studied the impact of financial crises on markets' efficiency. Particularly, they found that Asian crisis of 1997 adversely affected the efficiency of most stocks' markets, though markets' efficiency recovered to pre-crisis level. Ranking the markets' inefficiency, they found that markets were most efficient during the post crisis period, followed by pre-crisis period while during the crisis, markets high inefficiency.

Commenting the work conducted by previous researchers and by recruiting a *new* methodology for estimating the Hurst exponent, in this chapter we report how the Hurst exponent evolves over time and, particularly, we are interested in how financial crashes affect the behavior of S&P 500 index using *high frequency* data during 1996 - 2010 as it will be discussed in Section 4.

2.2 Irreversibility

Reversibility is a property of time series which, informally, states that if we reverse the time evolution of a time series its statistical properties do not change which means that the series do not contain asymmetries.

According to Zumbach (2009), a rigorous formulation of time reversal invariance is that the transformation $t \to -t$ is an exact symmetry of the system under consideration. A formal definition of time invariance is that a dynamic process $\{X_t\}$ is time reversible if for every positive integer n, the process $X = \{x_{t_1}, x_{t_2}, \dots, x_{t_n}\}$, and its time reverse process $X^- = \{x_{-t_1+m}, x_{-t_2+m}, \dots, x_{-t_n+m}\}$ have *asymptotically* the same joint distribution (Ramsey and Rothman, 1996). Based on this definition, reversibility requires stationarity, and since X and X^- are equally probable then, any random process is reversible. For instant, a linear Gaussian random process or even a monotonic transformation of a linear Gaussian random process are reversible while non-linear processes, non-Gaussian linear processes as well as linear ARMA models are time irreversible processes (Lacasa et al., 2012, 2015).

Generally speaking, financial markets are characterized by different types of asymmetry, i.e., *information asymmetry* between buyers and sellers, *perception asymmetry* which implies that not all investors react to news in the same manner, *behavioral asymmetry* meaning that price upswings are longer and slower than downswings, etc., and thus the financial time series are irreversible. Indeed, as stated in Ramsey and Rothman (1996), Zumbach (2009) and Xia et al. (2014), financial time series are inherently irreversible. Although, irreversibility is basically an all-or-nothing property meaning that a series is either reversible or not, we can also discriminate different degrees of time reversibility via the methodology described in Section 3.3. The closer the properties of an irreversible process are with a reversible process, the lower the degree of irreversibility is. Jiang et al. (2016) found that emerging markets are more time irreversible in comparison with developed markets and they have related predictability with irreversibility to some extent.

Studying the properties of the S&P 500 price-index from 1871 to 1988, Ramsey and Rothman (1996) found that the index series was irreversible and, in addition, the source of the irreversibility could be attributed to the underlying non-linear dependences. Further, Cox et al. (2005) claimed that "*irreversibility is the symptom of non-linearity*". Although financial time series are inherently irreversible, studying the local irreversibility Flanagan and Lacasa (2016) identified sub-periods where the properties of stock series could be characterized as reversible. This phenomenon led them to conclude that "*general stock price series are irreversible but periods of quasi-reversibility are not uncommon*". Moreover, they asserted that there was a strong link between irreversibility and predictability where higher degree of irreversibility indicated more predictable series. Studying 35 companies listed in the NYSE for the time interval from 1998 to 2012, they showed that the financial crisis of 2008 coincided with increasing values of irreversibility for all stocks.

Puglisi and Villamaina (2009) showed that memory acts as a hidden dissipative external force in a process and thus, its presence implies irreversibility. This assertion did not contrast with the findings of previously cited works which claimed that non-linear dependences were the source of irreversibility in the financial time series. As a final remark, we would like to point that Xia et al. (2014) showed that the presence of noise resulted a loss of irreversibility.

Therefore, quantifying the degree of irreversibility of a time series is actually an implicit way for quantifying the degree of non-linear dependences (memory) underlying in a time series, and consequently, the degree of predictability of a series.

In this chapter we study irreversibility as an alternative measure which can help us identify non-linear dependences among time series observations. This can be used as a complimentary method for quantifying index memory which will either validate the results obtained by calculating the Hurst exponent or give different results indicating the existence of a more complicated structure.

3. Visibility Graph Method for Hurst Exponent and Time Irreversibility

This section describes the method employed to calculate the Hurst exponent and quantify the time irreversibility of a series. Although statistics and econometric modeling are the most broadly used tools in economics and finance for analyzing data sets, there are cases that they do not perform well or the assumptions made do not correspond to reality. For these reasons, in this Section, we exploit the ability of methods proposed in the field of statistical mechanics to facilitate the analysis of the S&P 500 price-index.

3.1. Visibility Graph Approach

Visibility Graph (VG) is a novel method that maps a time series into a network according to a simple geometric criterion (Lacasa et al., 2008). It is shown that the associated graph inherits several properties of the initial time series and consequently, we can extract useful information regarding the properties of the initial time series by means of analysis conducted on the associated graph.

The geometric criterion that maps a time series into a network is mathematically rather simple. Let $\{X_t\}_{,t=1,2,..,n}$ be a series with n observations. Each observation of the original time series is transformed into a vertex in the associated network, and two random vertices i and j are "*visible*" to each other, if we can draw a straight line that connects the series data, and thus they are adjacent in the associated graph given the fact this line does not intersect any intermediate data height.

In Lacasa et al. (2008), there is a mathematically strict definition of VG method: Let $X = \{x_{t_1}, x_{t_2}, \dots, x_{t_n}\}$ be a time series with n observations. Any two arbitrary observations (t_a, x_a) and (t_b, x_b) will be adjacent in the associated graph if for any other observation (t_c, x_c) such that $t_a < t_c < t_b$ the following criterion is fulfilled:

$$x_c < x_b + (x_a - x_b) * \frac{t_b - t_c}{t_b - t_a}.$$

In **Fig. 2,** there is a graphical illustration of the VG method. A periodic time series with period four is depicted in the upper part of the figure while in the bottom we plot the associated graph extracted following the aforementioned procedure. Some remarks regarding the properties of the associated graph should be made before proceeding with explaining how we calculate the Hurst exponent through this method. Any visibility graph is always:

i. A connected graph since any arbitrary data series observation (t_a, x_a) is connected at least the previous one (t_{a-1}, x_{a-1}) and the following one (t_{a+1}, x_{a+1}).

ii. Invariant under affine transformation of series data.

In the next sub-section, the estimation of the Hurst exponent is illustrated by mapping a time series into a network.

3.2. Estimating Hurst Exponent

In the corresponding literature, there is a variety of methods for estimating the Hurst exponent, each of them with its own advantages and drawbacks. The most broadly used methods are the *detrending moving average*, the *detrended fluctuation analysis*, the *generalized Hurst exponent approach*, the *classical rescaled range analysis* and the *modified rescaled range analysis*. In our methodology, an approach recently introduced by Lacasa et al. (2009) is implemented, which overcomes many of the drawbacks that other methods have. Based on the discussion in the previous sections, our only assumption made is that the time series under study follow fBm.

Most of the properties of a network can be revealed by calculating its degree distribution. It is shown in Lacasa et al. (2009) and Ni et al, (2009) that fBm series map into scale-free networks with a power law degree distribution in form of:

$$P(k) = k^{-a},$$

where k is the degree of a node. It is also shown that there is a linear relation between the Hurst exponent (H) of a fBm and the exponent a of the degree distribution of the associated graph. The relation that binds H with a is:

$$H = \frac{3.1 - a}{2}.$$

Thus, the Hurst exponent of fBm can be estimated by mapping the series of the process into a network via the VG method and by calculating the exponent a of the degree distribution of the associated graph.

3.3. Quantifying Time Irreversibility

As it was described in Section 2, a time reversible process is the one whose properties remain invariant under time reversal. As shown in Lacasa et al. (2012), Donges et al. (2013), Lacasa and Flanagan (2015), Flanagan and Lacasa (2016), the VG method is capable of separating reversible and irreversible series as well as quantifying the degree of irreversibility for time variant series. This can be done by introducing the notion of VG irreversibility which implies that the properties of the associated graph remain invariant when we change the arrow of time.

In order to examine if a process is invariant under time reversal, first we have to introduce the notion of time in the associated graph. Thus, we divide the degree of a node k into in-degree k_{in} and out-degree k_{out} such that $k = k_{in} + k_{out}$. In-degree is the number of series data from the past that has visibility on a node i while out-degree is the number of future series data that a particular node i has visibility on.

According to Lacasa and Flanagan (2015), if a series is reversible then, the in-degree and the out-degree distributions should coincide asymptotically. If the process is irreversible then, the two distributions diverge and the higher the divergence, the more irreversible the series is. So, under these circumstances, a mathematic tool is implemented that can assist to examine whether the two distributions coincide and if not, then, to give a quantification of the divergence. In the context of this work, the Hellinger distance is used as a measure of distance between the two distributions.

Mathematically speaking, Hellinger (1909) distance between two discrete distributions $P(p_1, p_2, \dots, p_n)$ and $Q(q_1, q_1, \dots, q_n)$ can be calculated by the following formula:

$$H(P, Q) = \frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{n} (\sqrt{p_i} - \sqrt{q_i})^2}.$$

For reversible series of infinite length, the two distributions coincide asymptotically. In fact, none of real life series is of infinite length and thus we expect that the distance between the two distributions has a non-zero value even for reversible series, and as the length of the series increases, the two distributions converge asymptotically.

4. The Data

The dataset we use for the purpose of this research consists of tick data from Thompson Reuters Tick History (TRTH) for the S&P500 price-index starting on the January, 2^{nd} 1996 until June, 10^{th} 2010. Although, higher frequency leads inevitably to a larger microstructure noise, as it has been suggested by Hansen and Lunde (2006), we aggregate the data constructing a dataset that contains 5-minute data points which has more than 320,000 observations. In **Fig. 3**, we plot the daily closing prices of the index for the corresponding time interval. The red shadowed areas denote the two major financial crashes that happened between 1996 and 2010, while some other minor stock market crash incidents are also being marked.

5. Methodology

As it has already been described in Section 1, the micro-behavior of the S&P 500 price-index is investigated during the financial crises occurred. In order to accomplish this, we have to study the properties of the index series locally, i.e., to split the initial series into shorter overlapping periods of interest and study the properties of each sub-period (window). Conclusions can be drawn by comparing how properties of the index series vary over the course of time.

An important issue we have to address is to specify the length of windows. A long window will inevitably incorporate the impact of past events into the results which is not desirable, while a short window is probable to provide misleading results due to the small number of observations. Thus, in our case 10,400 observations are elaborated which correspond to 130 trading days or 6-month period. The number of increments between successive rolling windows is 240 observations or 3 trading days. Then, we apply the rolling window method into the index series to achieve continuity and for each window we estimate the values of Hurst exponent and calculate the degree of irreversibility via the VG method. In total there are 1,290 sub-periods of interest covering the period of January 1996 till June 2010.

At this point we specify the major crises that took place in US during the time interval 1996-2009. In **Table 1** the two major financial crises are reported.

Both financial crashes elaborated in this chapter are related to an index crash following a prolonged period of index rally. The beginning of the crisis is the day where the index reaches its maximum value before starting a downward sloping trajectory. On the other hand, the end of the crisis is considered as the day where the index reaches its lowest value before rebounding again.

In the next section, we present the empirical results obtained after processing with the data and the methodology described above.

6. Empirical Results

Hurst exponent is an index for quantifying the degree of persistency (or anti-persistency) of a time series and is broadly used for measuring the degree of efficiency in a market. The results of the analysis conducted on the original index series are presented in **Fig. 4** and **Fig. 5**. In **Fig. 4** we plot the values of the Hurst exponent for all sub-periods, while in **Fig. 5** the values of the Hellinger distance are plotted. The red shadowed regions in both plots denote periods of major financial crashes.

Compared with several other references (see Section 2 for more details), since our study contains high frequency data, the Hurst exponent inevitably varies significantly indicating that the properties of the index series are not constant but rather evolutionary over the course of time. We also report that only 4.4% of all windows calculated have an estimated Hurst exponent which lies within the interval $0.495 < H < 0.505$, i.e., it is very close to $H = 0.5$, and thus, we can assume that these sub-series have the same properties with a random walk. In total 49.16% of all sub-series exhibit persistency, while 46.44% can be characterized as anti-persistent. Actually, these results indicate that an fBm process is valid, since a high portion of sub-series studied appears to have an estimated Hurst exponent far different from $H = 0.5$. In particular, the maximum value of Hurst exponent is 0.773 and the minimum is 0.306, fact that exhibits that the Hurst exponent varies from extreme high to extreme low values. Furthermore, in **Fig. 4**, we observe that the Hurst exponent *"jumps"* from high to low values and opposite, which indicates that the behavior of the S&P 500index fluctuates between a high degree of persistency and anti-persistency. This also becomes visible if the summary statistics of the Hurst exponent are calculated. In this direction, in **Fig. 6** we plot the distribution of the Hurst exponent for all sub-periods. We report that the distribution is slightly right skewed and its tails are fatter than those of normal distribution. The mean value of the Hurst exponent is 0.501 and the standard deviation is 0.0873, which indicate the existence of large movements in the values of the exponent. This becomes obvious also in **Fig. 4**, where the Hurst exponent does not appear to follow any long-term trend but it fluctuates rather constantly around

$H = 0.5$. This observation exhibits the abrupt movements of investors' behaviors and these changes are mirrored in the index time series.

In Gençay at al. (2001), it was mentioned that stock market is a place where heterogeneous agents interact with each other while agents are distinguished mainly by the frequency they operate in the market. This result implies that by the use of high frequency data, we allow to integrate the behavior of all types of investors varying from fundamental to high frequency speculative investors. For instance, events such as the flash crashes that happened in May, 6th 2010 which was caused by crashes of high frequency trading algorithms and lasted for 30 minutes can also be captured in our analysis. Interactions among heterogeneous investors are the main reason why our results differ from those already presented in the corresponding literature. Although the results presented in **Fig. 4** are in line with Cajueiro and Tabak (2004a), who showed that the Hurst exponent was fluctuating around the value $H = 0.5$ for S&P 500 price-index series throughout January 1992 and December 2002, the Hurst exponent appears to be much more volatile in contrast with results presented in other papers using daily data (Cajueiro, Tabak, 2004a; Cajueiro, Tabak, 2004b; Cajueiro, Tabak, 2005; Grech, Mazur, 2004; Grech, Pamuła, 2008; Lim, 2007; Lim, Brooks, and Kim, 2008). Moreover, we do not observe prolonged trends on the plot of the Hurst exponent, but rather the value of the exponent fluctuates intensely.

These fluctuations become particularly obvious throughout the period 1996-2000, when the Hurst exponent reaches its' maximum values, $H > 0.65$, and then it drops abruptly losing more than 0.3 points in all cases. The same phenomenon took place at the end of 2006. It appears that high degree of persistency destabilizes the index series such that *highly persistent* periods are followed by *highly anti-persistent* periods.

Nevertheless, some more concrete patterns are observed in the local Hurst exponent series before the major index crashes. We observe that before both financial crises, i.e., Dot.com bubble crisis in 2000 and the Subprime mortgage crisis in 2007, the Hurst exponent peaks at its highest level, $H > 0.75$. This peak corresponds to the index boom that happened between the second half of 1998 and the first half of 1999 as well as between the second half of 2006 and first quarter of 2007 as shown in **Fig. 3**. The period of extreme increase in the index value is followed by a period where the index value is decreasing below 0.5 (high anti-persistency) before starting booming again. Generally speaking, anti-persistency indicates "*nervousness*" and we usually observe anti-persistency before shift points in the index series. Grech and Pamuła (2004) confirmed that Hurst exponent can be used as predictive tool of market crashes. In particular, they demonstrated that markets exhibit an anti-persistent behavior before the crash, while afterwards its' behavior becomes persistent.

About a year before each of those two crashes considered above, as it is observed from **Fig. 4**, the Hurst value fell temporarily below $H < 0.35$. However, after these two short periods, index series exhibit again an increasing persistency which was not as strong as it was before and it had a lower volatility. The onset of both financial crises is associated with quite prolonged periods of small fluctuations of the Hurst exponent at a level between $0.505 < H < 0.57$ indicating weak persistency. However, the index crash does not seem to alter investors' behavior in a long-term. What we observe is a short-living effect of the onset of the index crash and there is no sign of any repeated pattern throughout the crash.

In order to illustrate further that the Hurst exponent can be linked with index crashes, we have additionally marked with dash lines on **Fig. 1, 2** and **3**, some minor index crashes and we study the Hurst exponent throughout these crashes. The first crash happened on July 1998 and the second on May 2008. Even though the onset of both crashes finds the Hurst exponent lying in the anti-persistency band, it is followed in both cases by an increase in the Hurst exponent value which reaches the persistency band. Once again, an index crash is preceded by anti-persistency in the index series indicating "nervousness" in its investors' interactions.

Summarizing what we have discussed so far, except for the temporal effect that the onset of financial crisis has on the index behavior, the existence of financial crisis does not appear to affect considerably the properties of the index series. Stock market is an open dynamic system, which is adaptable on different disturbances happening in its environment and thus, the effect of most events has only short term duration.

In **Fig. 5**, we plot values for the Hellinger distance which gives a quantification of the degree of irreversibility of a series. As stated in Section 2, irreversibility is strongly related with the presence of non-linear dependences and memory, and consequently, by quantifying the degree of irreversibility, we can implicitly measure the degree of memory underlying in the series. Interestingly, the shape of the plot in **Fig. 5** is close to **Fig. 4**. This is not just a coincidence, but it means that the correlation between the Hurst exponent and the presence of memory in the series is high. In fact, except for the value $H = 0.5$, where increments of series are independent and uncorrelated, when $H \in [0, 0.5) \cup (0.5, 1]$, there is a form of dependence among increments and thus, a form of memory is also present. In particular, as we have already mentioned, when $H \in [0, 0.5)$, the time series under study can be characterized by long-term switching between high and low values in adjacent pairs while a value of Hurst exponent within the interval $H \in (0.5, 1]$ is an indication of a series which contain long-term positive autocorrelation.

The link between the Hurst exponent and the memory is also depicted in **Fig. 5**, where the irreversibility of the S&P 500 price-index series is studied. The correlation between Hurst exponent and Hellinger distance is 0.83, which means that higher (lower) values of Hurst exponent are associated with higher (lower) values of Hellinger distance. In particular, these results reveal that a persistent behavior of the index series is associated with a higher degree of irreversibility while anti-persistency is associated with lower levels of irreversibility. Based on what other researchers have stated on this topic (see Section 2.1 for more details), relating the level of irreversibility of a series with its predictability, higher levels of irreversibility exhibit higher degree of predictability and so, we have some evidences that sub-series whose Hurst exponent is $H > 0.5$ are more predictable than sub-series where $H < 0.5$.

7. Conclusions

In this contribution, the local properties of the S&P 500 price-index series are calculated as a way to investigate how the financial crashes that took place throughout the time interval from January 1996 to June 2010 have affected the behavior of the index series. By using high frequency data and the Visibility Graph methodology, the local Hurst exponent is estimated as a way to measure the persistency as well as to quantify the local degree of irreversibility of the S&P 500 price-index. We find that the properties of the index series vary over the course time and that both the Dot.com crisis in 2000 and the Subprime mortgage crisis of 2007 affected those properties only temporarily. Moreover, we provide evidence that the index series are linked with anti-persistent behavior before the crashes incident while after the onset of the crash the index series exhibit persistency. Irreversibility is in close relation with Hurst exponent which makes it a valuable tool that gives us an insight about the memory dynamics (linear and non-linear) underlying in a series and periods characterized as anti-persistent are found to contain less asymmetries.

Acknowledgement: The authors would like to acknowledge the gracious support of this work by the EPSRC and ESRC Centre for Doctoral Training on Quantification and Management of Risk & Uncertainty in Complex Systems & Environments (EP/L015927/1). This paper benefited from the comments of participants at the 2nd Quantitative Finance and Risk Analysis (QFRA) Symposium, Rhodes, Greece. Warm thanks are due to Prokopios Karadimos who read very carefully a preliminary version of our paper and who has afforded us con-

siderable assistance in enhancing both the quality of the findings and the clarity of their presentation.

Bibliography

Alvarez-Ramirez, J., Alvarez, J., Rodriguez, E. and Fernandez-Anaya, G. (2008). Time-varying Hurst exponent for US stock markets. *Physica A: Statistical Mechanics and its Applications*, 387(24), 6159 - 6169.

Bachelier, L. (1900). *Théorie de la spéculation*. Gauthier-Villars.

Barkoulas, J.T., Baum, C.F. and Travlos, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics*, 10(2), 177 - 184.

Barunik, J. and Kristoufek, L. (2010). On Hurst exponent estimation under heavy-tailed distributions. *Physica A: Statistical Mechanics and its Applications*, 389(18), 3844 - 3855.

Cajueiro, D.O. and Tabak, B.M. (2004a). The Hurst exponent over time: testing the assertion that emerging markets are becoming more efficient. *Physica A: Statistical Mechanics and its Applications*, 336(3), 521 - 537.

Cajueiro, D.O. and Tabak, B.M. (2004b). Ranking efficiency for emerging markets. *Chaos, Solitons & Fractals*, 22(2), 349 - 352.

Cajueiro, D.O. and Tabak, B.M. (2005). Testing for time-varying long-range dependence in volatility for emerging markets. *Physica A: Statistical Mechanics and its Applications*, 346(3), 577 - 588.

Campbell, J.Y., Lo, A.W.C. and MacKinlay, A.C. (1997). *The econometrics of financial markets* (Vol. 2, pp. 149-180). Princeton, NJ: Princeton University Press.

Clark, P.K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica*, 41(1), 135-155.

Cox, D.R., Hand, D.J. and Herzberg, A.M. (2005). *Foundations of statistical inference, theoretical statistics, time series and stochastic processes*. Cambridge University Press, 1st edition, UK.

Di Matteo, T. (2007). Multi-scaling in finance. *Quantitative finance*, 7(1), 21 - 36.

Donges, J.F., Donner, R.V. and Kurths, J. (2013). Testing time series irreversibility using complex network methods. *EPL (Europhysics Letters)*, 102(1), 10004.

Fama, E.F. (1963). Mandelbrot and the stable Paretian hypothesis. *The Journal of Business*, 36(4), 420 - 429.

Fama, E.F. (1965). The behavior of stock-market prices. *The Journal of Business*, 38(1), 34 - 105.

Fama, E.F. (1995). Random walks in stock market prices. *Financial Analysts Journal*, 51(1), 75 - 80.

Flanagan, R. and Lacasa, L. (2016). Irreversibility of financial time series: A graph-theoretical approach. *Physics Letters A*, 380(20), 1689 - 1697.

Gençay, R., Dacorogna, M., Muller, U. A., Pictet, O. and Olsen, R. (2001). An introduction to high-frequency finance. Academic Press, 1st Edition, California, USA.

Grech, D. and Mazur, Z. (2004). Can one make any crash prediction in finance using the local Hurst exponent idea? *Physica A: Statistical Mechanics and its Applications*, 336(1), 133 - 145.

Grech, D. and Pamuła, G. (2008). The local Hurst exponent of the financial time series in the vicinity of crashes on the Polish stock exchange market. *Physica A: Statistical Mechanics and its Applications*, 387(16), 4299 - 4308.

Greene, M.T. and Fielitz, B.D. (1977). Long-term dependence in common stock returns. *Journal of Financial Economics*, 4(3), 339 - 349.

Hansen, P.R. and Lunde, A. (2006). Realized variance and market microstructure noise. *Journal of Business & Economic Statistics*, 24(2), 127 - 161.

Hellinger, E. (1909). Neue Begründung der Theorie quadratischer Formen von unendlich-vielen Veränderlichen. *Journal für die reine und angewandte Mathematik*, 136, 210 - 271.

Hurst, H.E. (1956). The problem of long-term storage in reservoirs. *Hydrological Sciences Journal*, 1(3), 13 - 27.

Jiang, C., Shang, P. and Shi, W. (2016). Multiscale multifractal time irreversibility analysis of stock markets. *Physica A: Statistical Mechanics and its Applications,* 462, 492 - 507.

Kristoufek, L. (2010). On spurious anti-persistence in the US stock indices. *Chaos, Solitons & Fractals*, 43(1), 68 - 78.

Lacasa, L., Luque, B., Ballesteros, F., Luque, J. and Nuno, J.C. (2008). From time series to complex networks: The visibility graph. *Proceedings of the National Academy of Sciences,* 105(13), 4972-4975.

Lacasa, L., Luque, B., Luque, J. and Nuno, J.C. (2009). The visibility graph: A new method for estimating the Hurst exponent of fractional Brownian motion. *EPL (Europhysics Letters),* 86(3), 30001.

Lacasa, L., Nunez, A., Roldán, É., Parrondo, J.M. and Luque, B. (2012). Time series irreversibility: a visibility graph approach. *The European Physical Journal B*, 85(6), 1 - 11.

Lacasa, L. and Flanagan, R. (2015). Time reversibility from visibility graphs of nonstationary processes. *Physical Review E*, 92(2), 022817.

Lim, K.P. (2007). Ranking market efficiency for stock markets: A nonlinear perspective. *Physica A: Statistical Mechanics and its Applications*, 376, 445-454.

Lim, K.P., Brooks, R.D. and Kim, J.H. (2008). Financial crisis and stock market efficiency: Empirical evidence from Asian countries. *International Review of Financial Analysis*, 17(3), 571 - 591.

Malkiel, B.G. and Fama, E.F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2), 383-417.

Mandelbrot, B. (1967). The variation of some other speculative prices. *The Journal of Business*, 40(4), 393 - 413.

Mandelbrot, B. (1997). *The variation of certain speculative prices. In Fractals and Scaling in Finance* (pp. 371-418). Springer New York, USA.

Mandelbrot, B.B. and Van Ness, J.W. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM Review*, 10(4), 422 - 437.

Morales, R., Di Matteo, T., Gramatica, R. and Aste, T. (2012). Dynamical generalized Hurst exponent as a tool to monitor unstable periods in financial time series. *Physica A: Statistical Mechanics and its Applications*, 391(11), 3180 - 3189.

Ni, X.H., Jiang, Z.Q. and Zhou, W.X. (2009). Degree distributions of the visibility graphs mapped from fractional Brownian motions and multifractal random walks. *Physics Letters A*, 373(42), 3822 - 3826.

Osborne, M.F. (1959). Brownian motion in the stock market. *Operations Research*, 7(2), 145 - 173.

Puglisi, A. and Villamaina, D. (2009). Irreversible effects of memory. *EPL (Europhysics Letters)*, 88(3), 30004.

Ramsey, J.B. and Rothman, P. (1996). Time irreversibility and business cycle asymmetry. *Journal of Money, Credit and Banking*, 28(1), 1 - 21.

Samuelson, P.A. (1965). Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6(2), 41 - 49.

Zumbach, G. (2009). Time reversal invariance in finance. *Quantitative Finance*, 9(5), 505 - 515.

Xia, J., Shang, P., Wang, J. and Shi, W. (2014). Classifying of financial time series based on multiscale entropy and multiscale time irreversibility. *Physica A: Statistical Mechanics and Its Applications,* 400, 151 - 158.

Wang, Y., Liu, L., Gu, R., Cao, J. and Wang, H. (2010). Analysis of market efficiency for the Shanghai stock market over time. *Physica A: Statistical Mechanics and its Applications*, 389(8), 1635 - 1642.

TABLE 1

Sample period	
Sub-period 1 (Dot.com bubble collapse):	15th March 2000- 1st October 2002
Sub-period 2 (USA subprime crisis):	1st October 2007- 10th March 2009

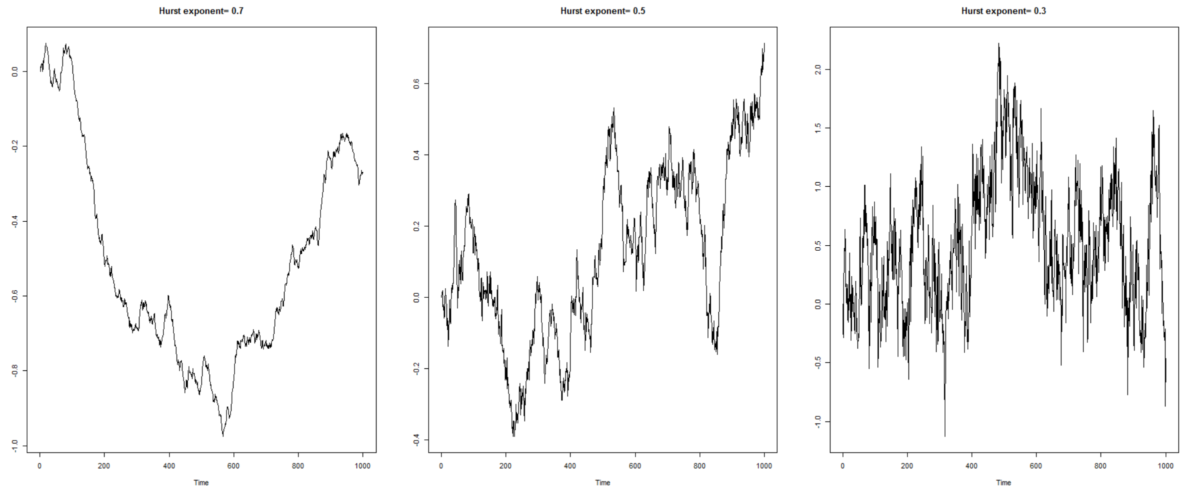

Figure 1. Fractional Brownian motion with (right to left) $H = 0.7$, $H = 0.5$ and $H = 0.3$, respectively.

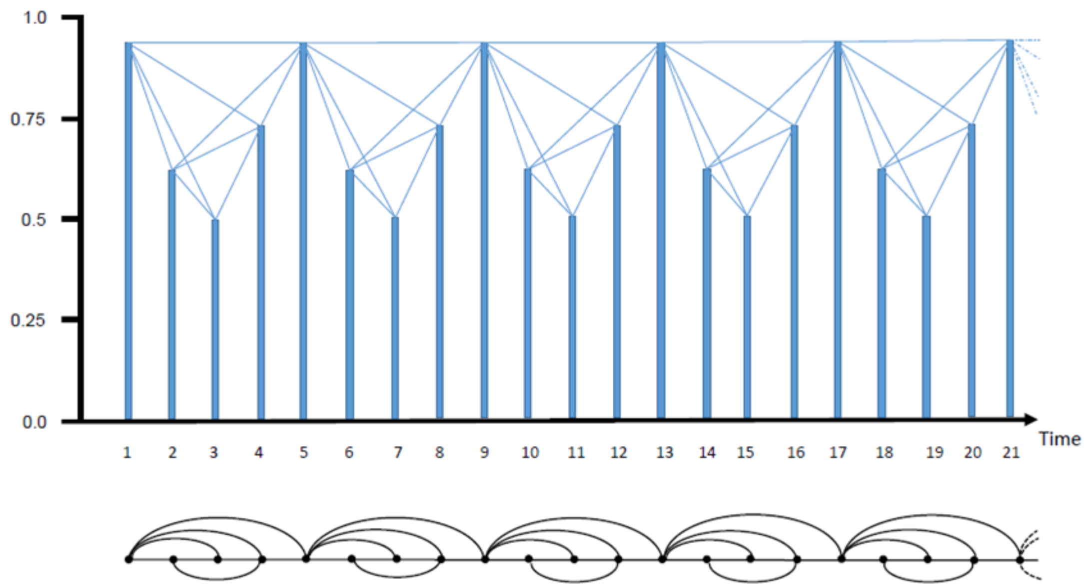

Figure 2. A graphical illustration of the Visibility Graph method. In the upper part there is a plot of a period time series while on the bottom it is plotted the corresponding Visibility Graph.

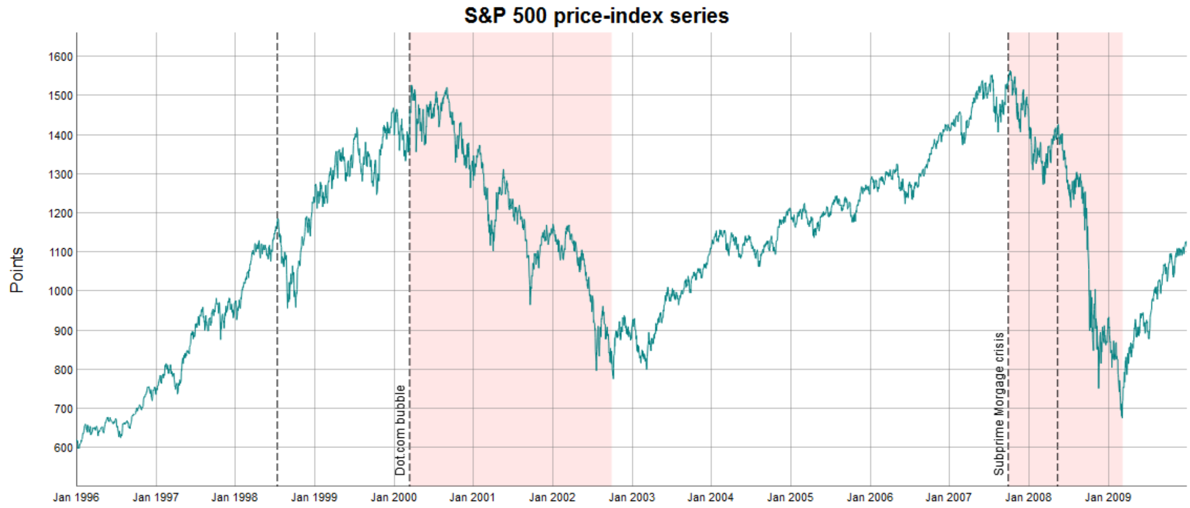

Figure 3. Plot of the daily closure points of the S&P 500 price-index for the period January 1996- June 2010.

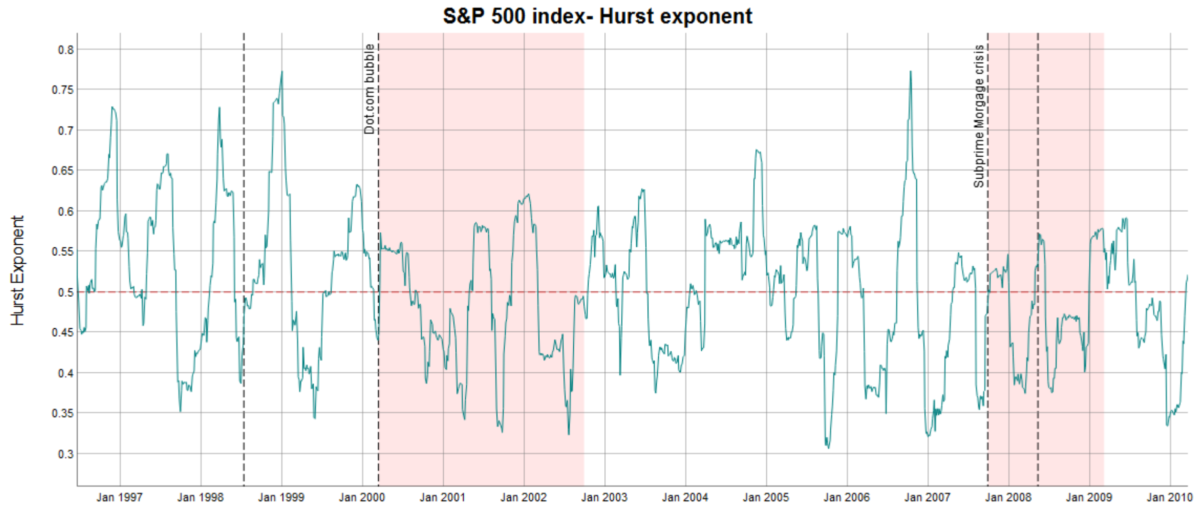

Figure 4. Plot of the estimated Hurst exponent for all windows. Red shadowed regions denote periods of financial crisis. The red line corresponds to the values $H = 0.5$ which characterizes the Brownian motion.

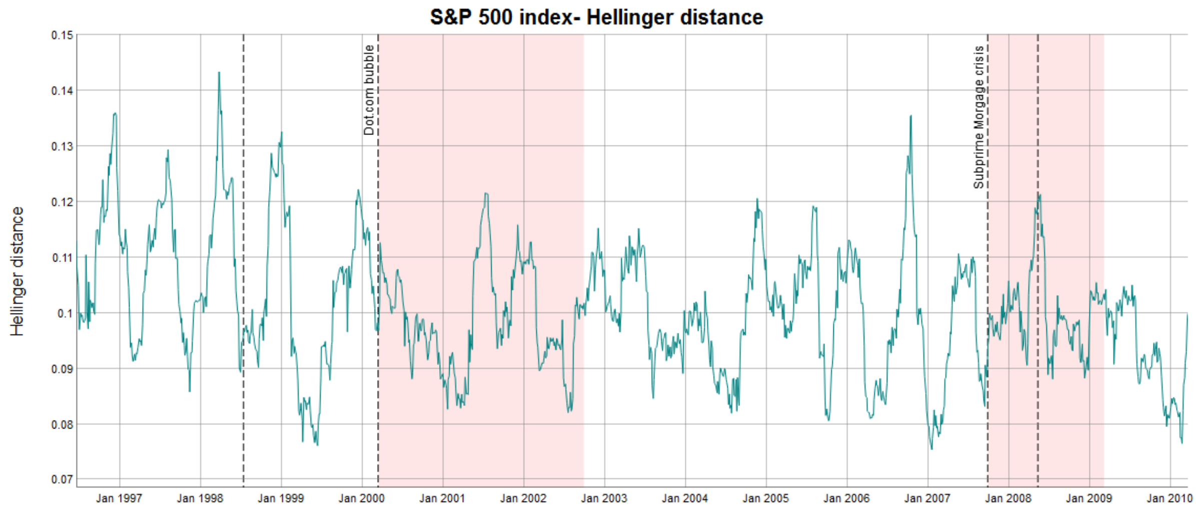

Figure 5. Plot of the Hellinger distance for all windows. Red shadowed regions denote periods of financial crisis.

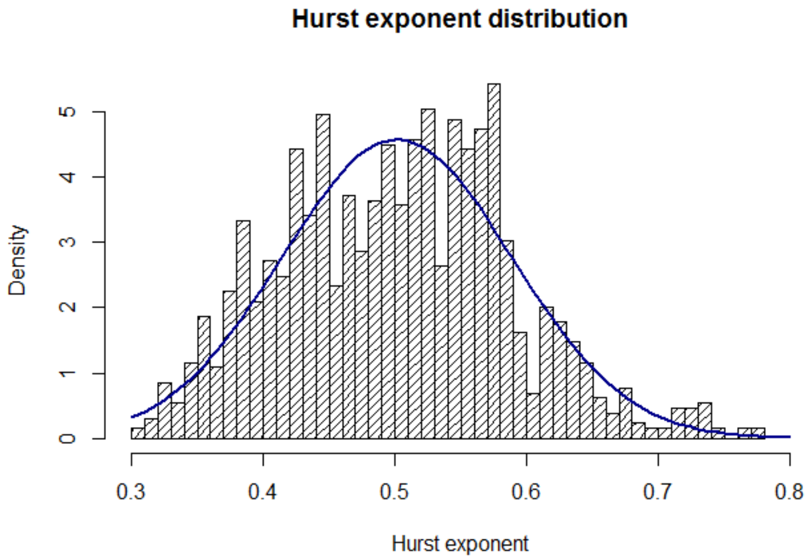

Figure 6. Distribution of Hurst exponent calculated over all 1290 sub-periods.