Ph225a **Problem Set #4 (Chapter II.6.)** November 3, 2004
(due date: November 17, 2004)

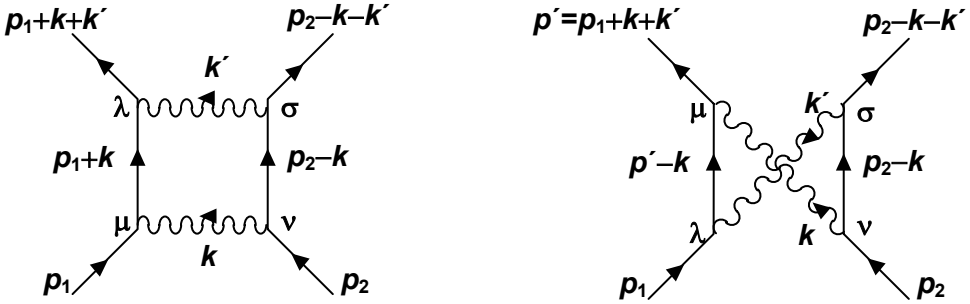
1. Consider Grassmann column vectors θ_i with i = 1, 2, …, N. Suppose that we make the transformation of variables such that $\phi_i = M_{ij}\theta_j$, where $\{M_{ij}\}$ denotes an $N \times N$ matrix with matrix elements being ordinary numbers, and we define

$$\int d\phi_1 d\phi_2 ... d\phi_N \left(\phi_1\phi_2 ... \phi_N\right) = \int d\theta_1 d\theta_2 ... d\theta_N \left(\theta_1\theta_2 ... \theta_N\right).$$

Prove that $$d\phi_1 d\phi_2 ... d\phi_N = \left\{\det[M]\right\}^{-1} d\theta_1 d\theta_2 ... d\theta_N ,$$

which is the opposite of the usual rule for differentials.

2. In our proof of gauge invariance for electrons scattering via exchange of photons, such as the two processes depicted in the Feynman diagrams below, we have only focused on one of the photons with momentum k, and have argued that other photons can in fact be treated like "spectators". To convince oneself about this statement, write down the complete amplitudes for the following two Feynman diagrams, and show that explicit consideration of the second photon of momentum k' indeed does not change the assertion that we have made in the class notes.



3. In our derivation of the anomalous magnetic moment of the electron, we have used the following form for the matrix element of the electromagnetic current:

$$\left\langle p',s'\left|J^{\mu}(0)\right|p,s\right\rangle = \overline{u}(p',s')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p,s)$$

where $q \equiv \left(p'-p\right)$, $\left|p,s\right\rangle$ denotes a state with an electron of momentum p and polarization s, and $F_1(q^2)$ and $F_2(q^2)$ are the form factors.

(a) Using Lorentz invariance and the current conservation, show that the form given above is indeed the most general allowed. [*Hint*: By Lorentz invariance, the right hand side of the above form must be a vector, which leaves the only possibilities as $\overline{u}\gamma^{\mu}u$, $\left(p+p'\right)^{\mu}\overline{u}u$, and $\left(p-p'\right)^{\mu}\overline{u}u$. The last term is ruled out because it is inconsistent with current conservation.]

(b) Show that all the following Feynman diagrams are proportional to $\overline{u}(p')\gamma^{\mu}u(p)$ and are therefore irrelevant to the magnetic moment.