**Ph225a** **<u>Problem Set #1</u> (Chapter II.1. and II.2.)** September 29, 2004

1. We have proven in class that the path integral representation for the amplitude of a free particle to propagate from a point $q_I$ to a point $q_F$ in time $t$ is given by

$$\left\langle q_F \,|\, e^{-iHt} \,|\, q_I \right\rangle = \int Dq(t') \exp\left[ i\int_0^t dt' \frac{1}{2} m\dot{q}^2 \right] ,$$

where $q(0) = q_I$ and $q(t) = q_F$, and $H$ denotes the free-particle Hamiltonian $H = \hat{p}^2/(2m)$, where $\hat{p}$ is the momentum operator. If we introduce a potential $V(\hat{q})$ to the Hamiltonian such that $H = \hat{p}^2/(2m) + V(\hat{q})$, prove that the new amplitude for a particle to propagate from $q_I$ to $q_F$ in time $t$ becomes:

$$\left\langle q_F \,|\, e^{-iHt} \,|\, q_I \right\rangle = \int Dq(t') \exp\left[ i\int_0^t dt' \frac{1}{2} m\dot{q}^2 - V(q) \right].$$

2. Find the quantity $\langle x_i\, x_j\, x_k\, x_l\, x_m\, x_n\rangle$ using the Wick's theorem, where $\langle x_i\, x_j\, x_k\, x_l\, x_m\, x_n\rangle$ is defined as:

$$\left\langle x_i x_j x_k x_l x_m x_n \right\rangle \equiv \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} dx_1 dx_2 \ldots dx_N \, e^{-\frac{1}{2}x\bullet A\bullet x} x_i x_j x_k x_l x_m x_n}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} dx_1 dx_2 \ldots dx_N \, e^{-\frac{1}{2}x\bullet A\bullet x}} ,$$

and $A$ denotes a real symmetric $N \times N$ matrix. (Hint: your answer should contain 15 different terms.)

3. In our derivation of the propagator $D_{\nu\lambda}(k)$ for a massive vector meson in QED, we arrive at

$$\left[-(k^2 - m^2)g^{\mu\nu} + k^\mu k^\nu\right] D_{\nu\lambda}(k) = \delta^\mu_\lambda ,$$

where $g^{\mu\nu}$ is the metric, (or the Minkowski tensor in flat space-time), $m$ is the mass of the meson, and $k$ is the four-dimensional momentum. Using $[D_{\nu\lambda}(k)]^{-1} = [-(k^2 - m^2)g_{\nu\lambda} + k_\nu k_\lambda]$, prove that

$$D_{\nu\lambda}(k) = -\left( g_{\nu\lambda} - \frac{k_\nu k_\lambda}{m^2} \right) \Big/ \left( k^2 - m^2 \right).$$