Multi-frequency Superconducting Cavity Stabilized Oscillators (SCSO) for Quantum-Gas Measurements and Gravitational Physics

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We report on the development of a superconducting cavity stabilized oscillator (SCSO) for use as a high-stability frequency source and for precision measurements. The SCSO system in its optimized condition has a potential frequency stability of parts in 10^{17} *to* 10^{18} *over short measurement times (up to* 10^3 *s). It can also operate in two resonant modes, which provides useful diagnostics of sources of frequency instability. This paper describes the progress of applying our SCSO to precise measurements of the equation of state of* ^{4}*He gas near* T_λ *and the concepts of using SCSO in conjunction with other ground and space clocks to perform tests of gravitational and relativistic physics.*

1. INTRODUCTION

Superconducting cavity stabilized oscillators (SCSO) have been known since the 1970's to provide the best frequency stability for short measurement times.[1] Using state-of-the-art microwave electronics,[2,3] improved temperature measurement and control,[4] and improved vibration isolation, the ultimate frequency stability of our SCSO system can achieve 10^{-17} to 10^{-18} for measurement times up to $\sim 10^3$ s. The SCSO system can be applied to precise measurements of the density and the equation of state of ^{4}He gas near the lambda transition (T_λ),[2,3] thereby providing insights into the nature of the interatomic interactions. In addition, SCSO can be developed into ultra-stable clocks for the advancement of gravitational and relativistic physics as well as other technologies (*e.g.* space navigation and global positioning system) that rely on stable clocks. We describe in the following the concepts and progress of utilizing SCSO for the aforementioned applications.

2. TECHNICAL DESCRIPTION

The SCSO utilizes state-of-the-art microwave electronics together with a high-Q superconducting niobium cavity in a phase-locked loop configuration to achieve high frequency stability.[2,3] To minimize fluctuations of the electromagnetic and mechanical properties of the cavity, our system will utilize high-resolution thermometry (HRT) for temperature control,[4] passive vibration isolation, and piezoelectric tilt correction. The cavity is a hollow cylinder of niobium which is chemically etched and then annealed at 1500 °C under 10^{-9} – 10^{-10} Torr for 24 – 48 hours,[3] yielding cavity Q of $\sim 10^9$ or higher at low temperature (~2 K), as exemplified in Fig. 1(a). The frequency stability of the SCSO system when optimized should be parts in $Q \times 10^7$.

Our SCSO can be operated in two resonant modes of the cavity, to wit, the TE_{011} and TE_{013} modes with respective frequencies of approximately 14 and 20 GHz. By monitoring the correlations of the two resonant frequencies as they change in time, many potential sources of frequency fluctuations ($\delta f/f$) can be determined because of their distinct frequency dependence.[5] For instance, thermal fluctuations yield $(\delta f/f)_{011}/(\delta f/f)_{013} \propto (f_{011}/f_{013})$ whereas mechanical vibrations result in $(\delta f/f)_{011}/(\delta f/f)_{013} \propto (f_{011}/f_{013})^{-2}$. Another potential source of frequency instability is the deviation of the cavity axis from the direction of gravity ($\delta\theta$). Any tilt in the cavity will cause mechanical distortions and therefore a frequency shift of $|\Delta f/f| \sim 10^{-11}(\delta\theta)^2$. To achieve our desired frequency stability of 10^{-17}, we require $\delta\theta < 10^{-4}$, corresponding to distances on the order of μm for our cavity dimensions ($\sim$ cm). The tilt adjustment can be provided by three piezo-electric tubes attached to the support plane of the cavity. By applying a ~kHz modulation voltage to one of the piezos, the other two piezos can be adjusted with a dc voltage to minimize the first and second modulation harmonics on the microwave resonant frequency of the cavity. Similar steps can be repeated on one of the other piezos to achieve optimal adjustments.

3. DENSITY MEASUREMENTS OF HELIUM-4 GAS

Our technique for the density measurements of helium utilizes the resonant frequency of the superconducting cavity, a technique similar to dielectric constant gas thermometry (DCGT). In the absence of gravity (see below), the frequency shift of the cavity caused by the presence of a dielectric

(ϵ) is given by $(\Delta f/f_0) = -\frac{1}{2}(\Delta\epsilon/\epsilon_0)$. The ability of our SCSO system to resolve frequency shifts to parts in 10^{15}, corresponds to equivalent resolution in the dielectric constant. This is in contrast to DCGT measurements using capacitors, which typically have resolution in ϵ of 10^{-9}.[6–8]

The dielectric constant (ϵ) is related to the molar density of the helium (ρ) via the Claussius-Mossotti relation, or to higher order in ρ:

$$(\epsilon - 1)/(\epsilon + 2) = A_\epsilon \rho \ (1 + b\rho + c\rho^2 + \cdots) \equiv \mu, \tag{1}$$

where A_ϵ is the molar polarizability, and b and c are the second and third dielectric virial coefficients, respectively, which may have temperature dependence. As we are unable to directly measure A_ϵ, b or c, we will assume published theoretical values as in, for example, Ref. 9.

An additional effect which we will correct for is the density gradient of the gas in the superconducting cavity caused by the Earth's gravity:

$$\rho(z) = \rho(z_0)\exp[-mg(z - z_0)/k_B T], \tag{2}$$

where z is the height in the cavity measured from some reference plane (z_0), m is the mass of a ^{4}He atom, g is the acceleration due to gravity, and k_B is Boltzmann's constant. If not accounted for, the density gradient would cause errors on the order of parts per million. Our cryogenic probe will have a reservoir of ^{4}He liquid below the superconducting cavity. The surface of the liquid can be measured by a capacitance level probe to a part in 10^8, which will define z_0 in the barometric formula above.

Assuming temperature resolution of parts in 10^{10}, pressure resolution of parts in 10^9 and frequency resolution of parts in 10^{15}, we can measure the density to parts in 10^{10} after correcting for gravity effects, which improves over the capacitor techniques with density resolution of parts in 10^6.[6,7] The temperature and pressure sensors will be calibrated against NIST standards.

3.1. Virial coefficients at low temperature

To characterize the pressure (p), temperature (T), and density data for the equation of state for ^{4}He gas, we use the virial equation:

$$p = RT\rho \ (1 + B\rho + C\rho^2 + \cdots), \tag{3}$$

where R is the molar gas constant. The second and third virial coefficients are functions of temperature given by B and C, respectively.

To access the virial coefficients experimentally, Eqs. (1) and (3) can be combined, yielding a standard equation of DCGT:[6]

$$p \ = \ A_1\mu \ (1 + A_2\mu + A_3\mu^2 + \cdots) \tag{4}$$

$$A_1 \equiv RT/A_\epsilon \tag{5}$$

$$A_2 \equiv (B-b)/A_\epsilon \tag{6}$$

$$A_3 \equiv (C-2Bb+2b^2-c)/A_\epsilon^2. \tag{7}$$

The virial expansion for ^{4}He for has been widely studied experimentally and theoretically. Considering the current interest in laser cooling and Bose-Einstein condensation of dilute atomic gas experiments, in which the interatomic interactions are weak, we hope to provide a counterpoint by studying a strongly interacting system. The virial coefficients describe the interactions between atoms in the gas. In particular, by studying ^{4}He vapor near the λ-point, we hope to gain insight into the superfluid state. At temperatures below 5 K, the uncertainties in modern *ab initio* computations of the second virial coefficient (at the part per thousand level)[9] now surpass the uncertainties in existing empirical data (at the percent level),[6,9] making it difficult to ascertain the validity of the various theoretical models. Using the experimental uncertainties listed previously, we will be able to measure B to parts in 10^5 precision for temperatures and pressures near the λ-point.

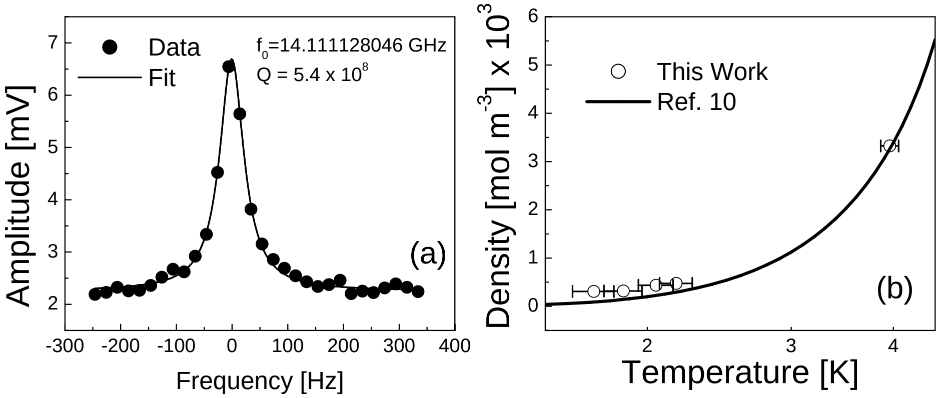

Fig. 1. (a) The amplitude vs. frequency (f) signal of the TE_{011} mode of one of our niobium cavities, showing $Q \sim 5.4 \times 10^8$ at 1.62 K. The solid line is a Lorentzian fit to the data. (b) Preliminary SCSO measurements of the density of ^{4}He saturated vapor as a function of temperature. The solid line represents data from Ref. 11. The errors in temperature are estimates based on two sensors nonlocal to the superconducting cavity and are uncertain due to thermal drifts. Improvements in temperature accuracy and stability can be achieved with calibrated sensors and HRT local to the cavity and with implementation of temperature control.

3.2. Saturated vapor equation of state near T_λ

The equation of state measurements can be further extended to include the saturated vapor line, particularly near the λ-transition. It is known that divergences in the thermal expansion parameter and the temperature derivative of the latent heat of evaporation occur at a few millikelvin above the superfluid transition temperature of ^{4}He.[10] With our improved density, temperature, and pressure measurements of saturated helium vapor near T_λ, we expect to gain new insight into the nature of the superfluid state. In particular, we plan to investigate the possible anomaly in the density of ^{4}He saturated vapor near T_λ, which may provide information for the interactions between the Bose condensed superfluid state and the vapor state.

We have performed preliminary measurements to demonstrate the concept of using the frequency shifts of SCSO to determine the density of saturated helium vapor. The results shown in Fig. 1(b) comprise measurements of the resonant frequency of both the empty cavity and cavity with saturated helium gas at 5 different temperatures corresponding to pressures of 30, 40, 70, 100 and 760 Torr. These preliminary data follow the trend of the NIST data[11], suggesting that the density of helium gas can be obtained from frequency measurements once calibrated temperature sensors are installed and analytical corrections to the density are made. The sensitivity of the preliminary density data is estimated at parts in 10^9, although the data lack corrections for the barometric effect and helium adsorption. The adsorption of ^{4}He onto niobium has not been studied adequately in the literature, so we will likely assume a behavior similar to ^{4}He adsorption onto another transition metal, such as copper.[8] We estimate the correction will be on the order of parts in 10^4. With further improvements in pressure readout and employment of HRT for fine temperature control, we expect to utilize the full capabilities of SCSO to obtain data of the ^{4}He equation of states with unprecedented resolution and accuracy.

4. CLOCKS FOR GRAVITATIONAL STUDIES

Some of the most scientifically important applications of high-stability frequency standards are tests of special and general relativity. Our SCSO system could be used in conjunction with other space- and ground-based clocks to test manifestations of Einstein's Equivalence Principle (EEP), such as the Local Position Invariance (LPI) and the Local Lorentz Invariance (LLI).[12] These measurements could be performed by linking our SCSO clock to a clock with good long-term stability, such as a cesium fountain clock, and then comparing with space-based clocks, such as PARCS[13] and SUMO.[14]

Another interesting application of our ultra-stable SCSO system is to detect gravitational radiation by Doppler tracking of spacecraft, as proposed by Vessot and collaborators.[15] A gravitational wave passing through a clock will cause a frequency shift in that clock. By placing ultra-stable clocks in a spacecraft and a ground station, one could look for unique gravity pulse signatures in the two-way and two one-way Doppler tracking data. Advanced software filtering techniques can isolate such a signature by suppressing correlated noises from known sources.[16] In addition, the multi-frequency capability of our SCSO can efficiently identify and correct for local frequency noise sources of each clock, as described in Section 2.

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