

Homework Assignment #4

(Due: **Fr 8 Nov 2019** / Please e-mail your work directly to ph118.homework@gmail.com)

I. Commercially-available, Low-Frequency, Low-Noise Amplifiers and their optimal use.

I have assembled spec sheets for the SRS family of low-noise amplifiers (attached). By showing this brand's offerings, I am not conveying my endorsement for these units. However, this vendor does provide a fairly extensive range of decent instrumentation amplifiers that I'll use as examples. Unfortunately, like most manufacturers, they do *not* provide all of the information you might like to have when planning for their use in your experiment! So, the potential user must be creative to assemble the information needed to plan an experiment with them. This is what this problem focuses upon...

- a) **SR550.** Note that the manufacturer does not state the current noise for the SR550.
 - i. From the information you can glean on the manufacturer's data sheet, combined with what you've learned in class, estimate the average value of the current noise. State your reasoning clearly, and also state what frequency range over which you believe your value should hold.
 - ii. With the value you estimated for the preamp's current noise, estimate R_{opt} at 1KHz, 100Hz, and 10Hz. State your logic.
 - iii. What is this amplifier's noise figure at 1KHz for a 10k Ω source impedance?
- b) **SR551.** The SR551 is advertised to be a "high impedance preamplifier".
 - i. Based on what you find on the data sheet, along with what you've learned in class, what kind of transistor do think is used in the front-end of this preamp?
 - ii. Note that the input capacitance for this preamp is not stated. In what measurement situations does input capacitance become important? Based on your previous answer to (i), research what are typical input capacitances for the type of device you believe is used in this preamp. Make sure your estimate is based on a device designed to operate in the frequency range of this amplifier.
 - iii. What is this amplifier's R_{opt} at 1KHz? Is this consistent with your previous answer to (i) and what you learned in class about the type of device you think is used?
 - iv. Give two examples of possible types of signal sources that would be optimally amplified by this unit. Given the input impedance stated in the manufacturer's data sheet and your answer to (ii) above, what concerns might you have when using this unit to amplify signals from the two signal sources you've identified?
- c) **SR552.** The SR552 has lower voltage noise than the previous two amplifiers.
 - i. At what "cost" does this lower noise voltage level come, i.e., what are the trade-offs to achieving such a low voltage noise spectral density?
 - ii. The current noise is not explicitly stated for this amplifier. From other information provided on the data sheet and what you've learned in class, deduce the current noise for this amplifier for a 1k Ω source impedance at 1kHz, 100Hz, and 10Hz. State your logic.
 - iii. Given the value you calculated above for the current noise, what is R_{opt} at 1kHz, 100Hz,

and 10Hz?

- iv. What is the amplifier's noise figure for a 1k Ω source impedance at 1kHz, 100Hz, and 10Hz?
- d) **SR554.** The SR554 has a built-in transformer input with a turns ratio of 100. This is followed by a transistor voltage amplifier with a gain of 5.
- i. The voltage noise for the transformer-coupled preamp is stated to be 0.1nV/ $\sqrt{\text{Hz}}$ RTI. Given what you learned in class, what kind of device is used for the first stage of the transistor voltage amplifier? State your logic.
 - ii. Assume the voltage noise is white from 10Hz up to the roll-off of this unit's response. Using data provided from the manufacturer and this assumption, what is the unit's current noise at 1kHz, 100Hz, and 10Hz?
 - iii. From info provided on the data sheet, along with what you've learned in class, estimate the value of the inductance for the transformer's primary winding.
- e) **SR560.** The SR560 is a variable-gain voltage amplifier.
- i. This unit's current noise specifications are not explicitly provided. From the information (on the data sheet) that is provided, deduce the amplifier's current noise at 1Hz, 10Hz, 1kHz, 100kHz, and 1MHz.
 - ii. From data deduced above, plus additional information provided by the manufacturer on the unit's data sheet, deduce the amplifier's R_{opt} at 1Hz, 10Hz, 1kHz, 100kHz, and 1MHz.
 - iii. From the values you deduced for R_{opt} , along with what you learned in class, what type of device is used as the front-end for this unit?

2. Noise expressions for the parallel-amplifier topology.

- I. The logic presented in f Lecture 9, led to the following stated, but as yet unproven, expression.

$$V_{n, RTI}^2 \Big|_{\text{N-amplifier system}} = V_{th}^2 + \left(V_{n,i} / \sqrt{N} \right)^2 + \left(\sqrt{N} I_{n,i} \right)^2 R_S^2$$

- a. Describe what all of the terms of this equation mean.
- b. Then prove the expression is, or isn't correct. (Please show all of your work, and state any assumptions you've made in deriving the expression.)

Clarification

In the slides for Lecture 9, I wrote down the expression:

$$V_{out}^2 = \sum_{i=1}^3 V_{out,i}^2 + \sum_{i \neq j} C_{ij} V_{out,i} V_{out,j}. \quad [1]$$

Perhaps the shorthand that I used was unclear to most of you; if so, sorry. Let me be more specific. First, we are talking about time-domain waveforms. Second, what I meant to imply was that Eq. 1 stems from the taking the mean squared value of the summed output.

Writing this explicitly, using more standard notation, it would look like this:

$$\langle V_{out}^2 \rangle \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} \left\{ \sum_{i=1}^3 V_{out,i}(t) \right\}^2 dt. \quad [2]$$

Parenthetically, note that a finite-time approximation to the true mean squared value of the waveform, integrated over an time interval T , would be

$$\langle V_{out}^2 \rangle_T \equiv \frac{1}{T} \int_{-T/2}^{+T/2} \left\{ \sum_{i=1}^3 V_{out,i}(t) \right\}^2 dt. \quad [3]$$

Accordingly, using clearer notation for Eq. 1 would make it look like this:

$$\langle V_{out}^2 \rangle_{total} = \langle V_{out,1}^2 \rangle + \langle V_{out,2}^2 \rangle + \langle V_{out,3}^2 \rangle + \sum_{i \neq j} C_{ij} \langle V_{out,i} V_{out,j} \rangle; \quad [4]$$

here, on the RHS, I have already begun to separate out direct terms from cross terms.

Your job in the homework problem is to look at all of these terms and argue which ones will survive a long-time average. And then you need to sum up those surviving terms to see if your answer agrees with mine.

3. Cross-correlation measurements: differentiating correlated from uncorrelated signals/noise.

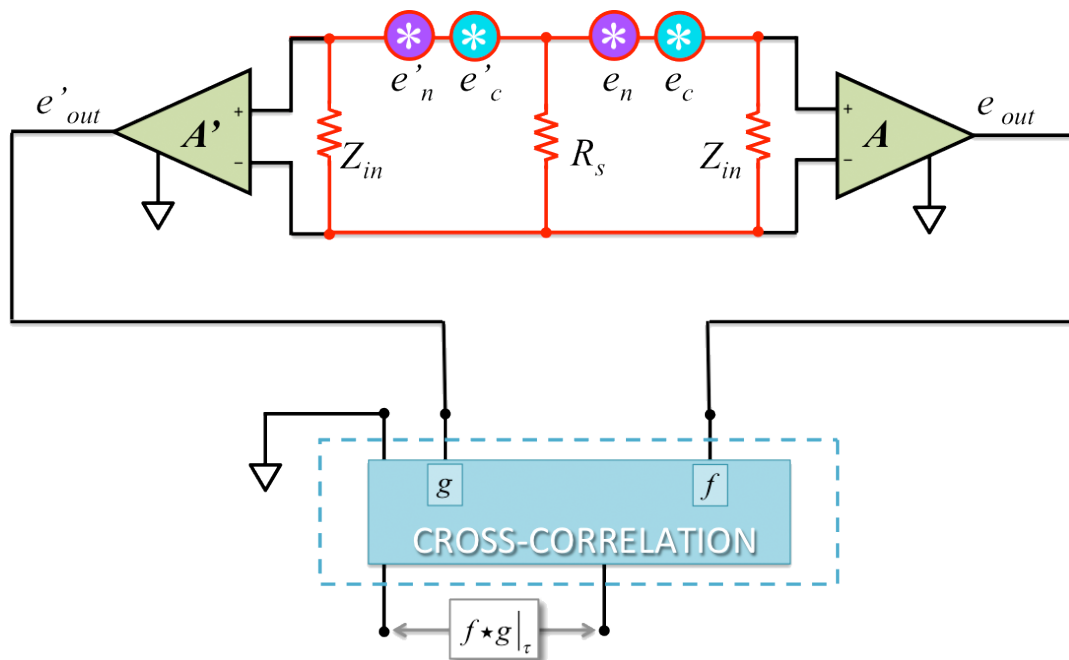
In the notes for Lecture 10, I conveyed the basics about cross-correlation measurements, using the simplest example of a two-amplifier system. I concluded by stating that the only terms which “survive” the cross-correlation operation are the following:

$$v \star v' \Big|_C \sim e_S \star e_S + \left(e_n \star i_n + e'_n \star i'_n \right) R_S + \left(i_n \star i_n + i'_n \star i'_n + i_{th} \star i_{th} \right) R_S^2$$

- a) Did I do the math correctly, i.e., do you agree? Check my answer by deriving all of the terms that arise in the expression:

$$v \star v' \equiv v \star v' \Big|_C + v \star v' \Big|_U$$

- b) In the expression above, the subscripts C and U denote the correlated (surviving) and uncorrelated (suppressed) terms in the cross correlation. Please note that here, and in the notes, I have suppressed an argument, τ , that should appear – the delay in the cross-correlation operation. What is the physical meaning of the delay, τ ?
- c) You have now derived all of the terms in the expression for $v \star v'$. Argue which of these terms are suppressed by cross-correlation – and please make sure to state your logic (i.e., don't just say something superficial like “they vanish”.)
- d) An astute member of a previous class has speculated that with finite, and not infinite, amplifier input impedance the voltage noise from one channel could possibly appear at the other – and, hence, would survive in the cross-correlated output. To help you analyze whether this is true or not, consider the following circuit. I have only included the voltage noise generators from the amplifiers for clarity. Have I drawn it correctly? If not, please redraw it correctly.



- e) Using what you perceive to be the correct circuit diagram, consider the voltage noise e_n from the “unprimed” amplifier. Does it survive the cross-correlation operation? If so, please provide an expression for the term that appears at the output of the cross-correlator.