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Homework Assignment #3 (*…revised*)

(Due: Monday, 28 Oct 2019 / Please e-mail your work directly to rmkatti@caltech.edu)

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1. Noise performance, gain, and dynamic range of an amplifier chain.

I have the collection of amplifiers below which all have 50Ω input and output impedances.

Amplifier 1: Max output=33dBm, BW=1MHz, $|H|^2$ = 20 dB, $\sqrt{S_\Sigma}$ = 8.0 nV/√Hz

Amplifier 2: $|H|^2$ = 30 dB, S_Σ = -179 dBm/Hz, V_{max}(out) = 0.707 V(rms), BW=100kHz

Amplifier 3: BW=1MHz, $|H|^2$ = 10 dB, 200mW max. output, V_Σ(RTI) = 3.0 μV rms

Amplifier 4: *NF*=7.0dB, *DR* = 116 db, BW=100kHz, $|H|^2$ = 15 dB

Here, BW is bandwidth, $|H|^2$ is the forward (power) transfer function (assume it's white over the passband), DR is the dynamic range, and $\sqrt{S_\Sigma} = \sqrt{S_\Sigma}(R_S)$ is the total amplifier voltage spectral density at for source impedance R_S.

a) Arrange the amplifiers in correct order to maximize performance of the cascade. Please explain your logic.

b) What is the NF, BW, DR, and gain of the resulting cascade?

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2. Equivalent expressions for noise.

a) Write an expression for S_{th}, the thermal noise power spectral density available from a resistor at temperature T to a matched load, in dBm/Hz.

b) What are the values of thermal noise spectral density at 293K, 78K, and 4.2K, in dBm/Hz?

Invariably, the units dBm are used in the context of 50Ω systems.

c) Write an expression relating voltage, V, to power P (in dBm).

d) In class we modeled an amplifier's total equivalent noise as a being equivalent to a generator with power spectral density S_Σ, coupled to the source resistance R_S; recall that this included both the amplifier's voltage and current noise. Write an expression relating S_Σ, in dBm/Hz, to the amplifier's noise figure (NF) in dB. (Note that *NF* assumes a white spectrum, so it is OK to write your answer, as a noise spectral density S_Σ.) Remember, also, that *NF* assumes the reference source resistor is at room temperature, T=293K.

e) Invert the expression in (d): write an expression relating for noise figure (NF) in dB as a function of noise power spectral density, S_Σ, in dBm.

f) In the answer you get for (e), what value of S_Σ in dBm/Hz must you plug in to get a 0dB *NF*? Does that make sense?

3. Home-made seismometer.

Scenario: I am at Peet's with a latte, a pencil, paper, and my smartphone – and I am daydreaming about making a crude seismometer with my son. (This is an example of how one can go about making a sensitivity estimate of a measurement apparatus that one might like to build.)

First, let's consider what the nature of the raw signal to be detected.

I went to the web and learned the following:

"The movement during fault rupture produces a range of vibrations, or seismic waves, that are radiated outwards from the quake's epicenter. The vibrations of engineering significance occur at frequencies from less than 0.2 Hz to 20 Hz (periods from about 5 seconds down to about 0.05 seconds)."

Okay so at first pass let's assume the signal will be a motional sine wave at 10Hz.

Elsewhere on the web I also found that:

"Originally earthquake magnitudes were based on the amplitude of ground motion displacement as measured by a standard seismograph. The best known of these is the Richter Magnitude which was defined for local earthquakes in southern California.

ML (Richter Scale) = logδx + 2.56logD - 1.67

Where δx is the measured ground motion (in μm) and D is the distance from the event (in km). This is still used for measuring the magnitude of shallow events at distances less than 600 km (today called the Local Magnitude)."

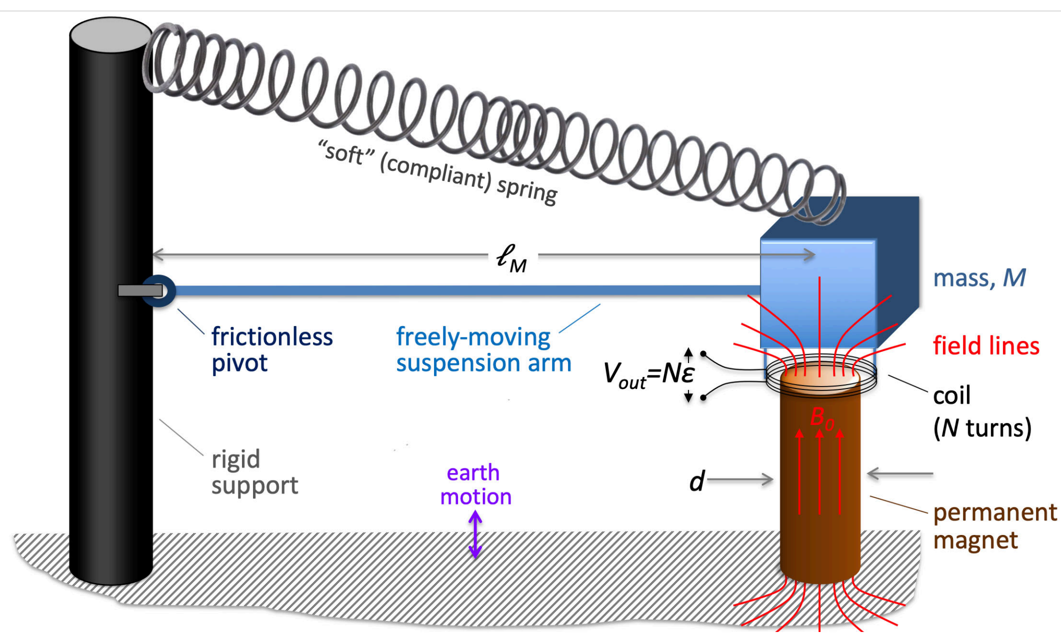
Okay, so solving this for δx:

$$10^{ML} = 10^{log\delta x + 2.56 logD(km) - 1.67} = \delta x D^{2.56}/10^{1.67}$$

$$\delta x(\mu m) = (46.8)\, 10^{ML}/D(km)^{2.56}$$

So, let's say that we want to be able to see local earthquakes 100km away that are at least magnitude 3 and higher on the Richter scale. For this case, I calculate a peak displacement of about δx ~ 0.360μm. Please check to see that I've done this correctly!

We'll make a simplest implementation – a *moving coil seismometer*. Here's a simple sketch:



The windings are attached to the ("proof") mass on distal end of a freely-moving suspension arm. This is free to move in the vertical direction, by means of the frictionless pivot on the rigid vertical support bar. We should assume that both the support bar and the magnet, which are anchored to the ground, move up and down with earth motion during a quake. Given the soft-spring suspension, the big proof mass will remain roughly stationary since it is, in effect, vibration isolated. (One must ensure that the resonant frequency of the mass/spring system is lower than lowest signal to be detected. In this case, the mass/spring system behaves as a low-pass vibrational filter and attenuates transmission of vibrational forces at frequencies above resonance.) The detection coil, which is rigidly attached to the proof mass, remains essentially stationary ("floats" with the proof mass) as the permanent magnet moves with the earth.

The magnet, which we'll assume has a diameter, *d* = 2cm has maximum B-field at its ends and then drops off with distance from the end. So, when the coil moves there is a time-dependent flux change through the coil, and this generates an EMF, ε.

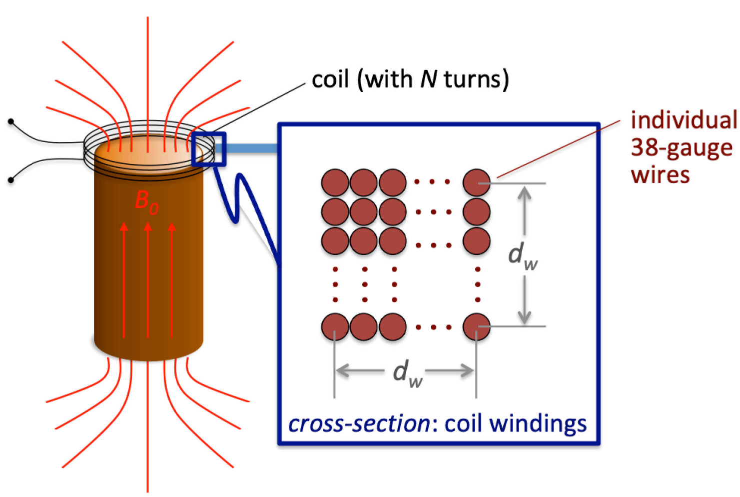
We can maximize the induced voltage by winding many turns of the coil; this multiplies the EMF per-turn, ε, to yield $V_{out} = N\varepsilon$. Here, N is the number of turns. (This enhancement holds if the coil is small enough that the flux is roughly constant for each turn.)

See if you agree with my derivation of the relation between ground motion and open-circuit output voltage. I believe it can be approximated as:

$$V_{out}(rms) = N\left(\frac{B_0}{d}\right)\left(\frac{\omega\delta x}{\sqrt{2}}\right)d^2$$

To arrive at this, I've made the following very crude approximations (I'll call them "*espresso joint approximations*" from now on!):

- The field gradient at the very end of the magnet (where it's strongest) is of order $\frac{\delta B}{\delta x} \sim \frac{B_0}{d}$. Here, B_0 is the peak magnetic field of the magnet. For a so-called "high energy product" magnet, such as obtained with a samarium cobalt alloy, $B_0 \sim 1T$.
- The cross section of the coil should be only a fraction of the magnet's diameter to ensure that minimal flux is lost *through* the windings (we want it to go through the center of the coil and do the work of generating an EMF). I therefore will assume the windings have cross section d_w = 0.1*d*, and that there is a spacing of 0.5*d* on either side of the coil, so that it doesn't rub on the magnet as it moves up and down. Hence, the coil diameter should be d_{coil} = 1.15*d*. Here's a little sketch to ensure you're not confused by my wording:



Okay, let's move on to envisioning the actual construction of our seismometer. I've got a bunch of enamelinsulated 38-gauge copper wire at home. It has a diameter of about 100μm and, checking on my smartphone, I find it should have a resistance per unit length of about 660Ω/1000ft.

Okay, now to assess how this contraption will perform…

Show all of your derivations for the questions below:

a) Determine *N,* the number of turns can I fit into the coil cross section. Assume close packing.

b) What is the rms magnitude of the voltage generated by the seismometer coil for a magnitude 3.0 earthquake with an epicenter 100km away?

c) What is the coil resistance, R_{coil}? What is the magnitude of its thermal noise at room temperature?

d) Assume I make a transformer-based impedance transformation to match R_{coil} to R_{opt} for a PAR113 preamplifier. What sort of voltage boost will I get from this transformation? (For reference: The background for this is covered in Lecture 7.)

e) Using the transformer-coupled PAR113 readout, what is the minimum ground motion I can resolve? (Assume that "resolve" means achieve SNR=1.) Here, assume the measurement bandwidth spans the aforementioned range: 0.2Hz to 20Hz. What limits the sensitivity of the apparatus?

f) What is the smallest (Richter-) magnitude earthquake with an epicenter at a distance of **D=600km** that I can resolve with my contraption?