Failures in Power-Combining Arrays

David B. Rutledge, Fellow, IEEE, Nai-Shuo Cheng, Robert A. York, Member, IEEE, Robert M. Weikle II, Member, IEEE, and Michael P. De Lisio, Member, IEEE

Abstract—We derive a simple formula for the change in output when a device fails in a power-combining structure with identical, matched devices. The loss is written in terms of the scattering coefficient of the failed device and the reflection coefficient of an input port in the combining network. We apply this formula to several power combiners, including arrays in free space and in enclosed waveguide structures. Our simulations indicate the output power degrades gracefully as devices fail, in agreement with previously published results.

Index Terms—Quasi-optics, grid arrays, power combining, graceful degradation.

I. INTRODUCTION

QUASI-optical arrays use periodic structures loaded with semiconductor devices for power combining. The behavior of these arrays when devices fail is extremely important. The limited experimental data that have been published suggest that the arrays will degrade gracefully as devices fail [1], [2]. In this paper, we develop a simple formula for calculating the loss when a device fails. We then apply this formula to several power-combining arrays, and compare our simulations to measured results.

Graceful degradation has been studied previously in more traditional power-combining systems. Ernst, Camisa, and Presser [3] used an intuitive approach to characterize the graceful degradation of power-combined amplifiers. Saleh [4] used this result in a paper proposing a combining scheme with improved graceful degradation. Sarkar and Agrawal [5] used Kurokawa’s theory of multiple-device oscillators to predict the graceful degradation of coupled oscillators.

This work was supported by the U.S. Air Force Material Command/Rome Laboratory and by the U.S. Army Research Office under a MURI grant to Caltech.

D. B. Rutledge is with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125.

N.-S. Cheng and R. A. York are with the Department of Electrical and Computer Engineering, University of California at Santa Barbara, Santa Barbara, CA 93106.

R. M. Weikle is with the Department of Electrical Engineering, University of Virginia, Charlottesville, VA 22903.

M. P. De Lisio is with the Department of Electrical Engineering, University of Hawai`i at Mānoa, Honolulu, HI 96822.

II. THEORY

Consider a power combiner with $N$ identical sources and a load. The device-device and device-load coupling in the circuit are described by a scattering network $\bar{S}$ with $N+1$ ports, as shown in Fig. 1. A wave equivalent circuit represents each source with a wave $c$ and a scattering coefficient $s_g$. The load is a one-port device with a reflection coefficient $s_l$. We choose normalizing impedances at the device and output ports such that $s_g$ and $s_l$ are zero. We call the output port 0 and use the index $i$ for the source ports 1 through $N$. We can write the output wave $b_0$ as

$$b_0 = \sum_{i=1}^{N} s_{0i} c. \quad (1)$$

We now make two assumptions. First, we assume that in normal operation all of the input ports are matched. This means that the scattered waves $b_1, \ldots, b_N$ are zero. For an input port $j$, we write

$$b_j = \sum_{i=1}^{N} s_{ij} c = 0. \quad (2)$$

This expression can be rewritten as

$$s_{jj} = -\sum_{i \neq j} s_{ji}. \quad (3)$$
Our second assumption is that the coupling of each of the sources to the output load is the same. That is, all $s_{ii}$ are identical. In order to conserve power under normal operation, we demand
\[ |s_{ii}|^2 = \frac{1}{N}. \tag{4} \]

Now consider the effect when source $j$ fails and no longer produces a source wave. We assume the other sources do not change. We modify (1) to write the new output wave $b'_0$ as
\[ b'_0 = \sum_{i=1, i\neq j}^{N} s'_{ii} c, \tag{5} \]

where the coefficients $s'_{ii}$ are the scattering parameters of the $N$-port network formed by the original network $S$ and the failed device. These new scattering parameters can be written in terms of the original scattering parameters and the reflection coefficient $\rho$ of the failed device using Mason’s rule [6]
\[ s'_{ii} = s_{ii} + \frac{s_{ij} s_{ji} \rho}{1 - s_{jj} \rho}. \tag{6} \]

We now substitute this expression into (5) and express the output wave as
\[ b'_0 = \sum_{i=1}^{N} s_{ii} c + \sum_{i=1, i\neq j}^{N} \frac{s_{ij} s_{ji} \rho c}{1 - s_{jj} \rho}. \tag{7} \]

Finally, using (1) and (3) we can rewrite this as a proportional change in the output wave:
\[ \frac{b'_0 - b_0}{b_0} = -\frac{1}{N(1 - s_{jj} \rho)}. \tag{8} \]

This is a simple formula to apply if we can determine $s_{jj}$, the reflection coefficient for an input port in the combining network. In traditional power combining circuits with hybrids or corporate combiners, the network $S$ is usually matched, so that $s_{jj}$ is zero [7]. For these circuits, the proportional change in output wave is $-1/N$. For large $N$, the fraction of power lost is approximately $2/N$.

As long as our original assumptions are met, equation (8) can be iterated for multiple failures. Two special cases are worthy of note. The first is if a device always fails as a matched load ($\rho = 0$). The second case is if the inputs are matched and noninteracting, guaranteeing that $s_{jj} = 0$ regardless of any previous failures. If either of these cases are true, iterating (8) gives a particularly simple result for multiple failures:
\[ \frac{b'_0 - b_0}{b_0} = -f, \tag{9} \]

where $f$ is the fraction of failed devices. We can recast (9) to give the reduction in the output power:
\[ \left| \frac{b'_0}{b_0} \right|^2 = (1 - f)^2. \tag{10} \]

These expressions are consistent with the idea that each device contributes equally to the output wave, and agree with the results derived in [3] and [5]. Because of their simplicity, (9) and (10) will be considered archetypal failure expressions.

One limitation in our approach is that it assumes that a failure does not change the other sources in the network. This means that the bias on the remaining devices should not change, and that their power or frequency should not vary. Furthermore, for array amplifiers, mixers, or detectors, we would need to consider how the input power to the array would change when a device fails.

\section*{III. DISCUSSION}

We will first consider two very simple examples. Fig. 2 shows a square grid array with the sources connected in parallel along a row, and with the rows in series. The load resistance is the same as the resistance of an individual source. This simple model is most useful for mixer and detector grids [8], [9], where a microwave signal is downconverted to a much lower frequency. This model is less useful for oscillator and amplifier arrays because it does not account for any phase delays or transmission-line effects which must be present in grids that are comparable to a wavelength. Nevertheless, for a large grid, the load that a single generator sees is primarily the parallel combination of the other sources in that row. We can approximate $s_{jj}$ as
\[ s_{jj} \approx \frac{2}{\sqrt{N}} - 1. \tag{11} \]

If a device fails as a short circuit, $\rho$ is $-1$ and we can approximate the proportional change in output wave as $-1/(2\sqrt{N})$, and the fraction of power lost as $1/(\sqrt{N})$. This is bad, because the failed device shorts out the entire row. The loss is less when the device fails as an open, or when a shorted device can be fused. In this case, $\rho$ is $+1$ and the proportional power reduction is $1/N$. This is the best possible situation with a single failure.

We can reduce the sensitivity of the grid to $\rho$ if we connect horizontal resistors between the columns, as shown in Fig. 3. The resistance in each of the horizontal connections is the same as the individual source and output resistance. For a large grid, calculating $s_{jj}$ away from the edges is an old problem that has appeared in many homework questions and exams over the ages. One can use superposition to show that the resistance shunting any resistor in an infinite grid of identical resistors is the resistance of a single resistor. This makes $s_{jj} \approx 0$, and the proportional power loss $2/N$.

The loss is twice as large as in the previous example, but no longer depends on the impedance of the fault.
Fig. 2. Simplified model of a square grid. All resistors are equal.

Fig. 3. Simplified model of a square grid with resistors in the horizontal lines. All resistors are equal.

Fig. 4. A grid array of dipole antennas in a hard-wall waveguide.

IV. ARRAYS IN WAVEGUIDE

We can extend our development to investigate some more realistic power combiners. Many researchers believe that quasi-optical arrays will be most useful when placed in “hard-wall” waveguides, like those reported in [10]–[12]. These waveguides are large, overmoded structures with dielectric loading on the sidewalls to simulate a perfect magnetic conductor. This waveguide will support a quasi-TEM mode over a certain bandwidth. Another promising approach demonstrated by York and co-workers [13] is to combine the power in a standard waveguide.

We will simulate an array of dipole antennas placed in an idealized hard-wall waveguide. Fig. 4 shows the idea. A periodic square array of $N$ dipoles exists in a waveguide with electric walls on the top and bottom, and magnetic walls on the sides. The antennas are separated by a distance $u$, and the size of the waveguide $a$ is thus $\sqrt{N}u$. The waveguide extends to infinity the $x$-direction. The inputs are the $N$ antennas, and the output is the TEM waveguide mode. Our goal is to determine the scattering matrix $\mathbf{S}$ of this $(N + 1)$-port structure. Note that the matrix $\mathbf{S}$ will not necessarily be lossless because some power may propagate in the waveguide’s higher-order modes.

We compute the scattering matrix $\mathbf{S}$ as follows. The self impedance of any dipole in the waveguide can be calculated using the induced EMF technique. This method has long been used in analyzing quasi-optical grids [14], [15]. The induced EMF technique can also be used to determine the mutual impedance between any two antennas in the array. We begin by expressing the assumed $y$-directed surface current distribution on antenna $i$ as a Fourier series:

$$K^i(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} K^i_{mn} \cos \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{a} \right).$$

The Fourier coefficients $K^i_{mn}$ can be determined using the standard inversion exploiting the orthogonality of the cosine function. This current will induce a $y$-directed electric field $E^i(x, y)$. We now employ the induced EMF technique [16] to find the mutual impedance between two antennas $i$ and $j$:

$$z_{ij} = -\frac{1}{I_i I_j} \int_0^a \int_0^a E^i(x, y) K^j(x, y) dx dy$$

$$= \frac{a^2}{I_i I_j} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \epsilon_m \epsilon_n K^i_{mn} K^j_{mn} Z_{mn}. $$

$$\text{(13)}$$
$I_i$ and $I_j$ are the rms terminal currents flowing into antennas $i$ and $j$. $e_m$ is Neuman’s number and is equal to 1 for $m = 0$ and is equal to 2 for all other $m$. $Z_{mn}^{\text{eff}}$ is an effective mode impedance given by

$$Z_{mn}^{\text{eff}} = \frac{m^2(Z_{mn}^{\text{TE}}/2) + n^2(Z_{mn}^{\text{TM}}/2)}{m^2 + n^2}. \quad (14)$$

$Z_{mn}^{\text{TE}}$ and $Z_{mn}^{\text{TM}}$ are the TE and TM mode impedances of the waveguide. In a more general case these impedance terms may be replaced by parallel combinations of mode impedances seen looking in the positive and negative z-directions as in [17].

After computing the mutual impedance between all possible antenna pairs, we will have determined an $N$-port impedance matrix $\bar{Z}$. This impedance matrix is used to relate

\begin{table}[h]
\centering
\caption{Simulated Effects of a Single Device Failure in a 9-Element Waveguide Dipole Array}
\begin{tabular}{|c|c|c|c|}
\hline
Fault Location & Output Reduction (dB) \\
\hline
(column, row) & Match & Open & Short \\
\hline
(1, 1)$^-$ & 1.02 & 0.99 & 1.05 \\
\hline
(1, 2)$^+$ & 1.02 & 0.71 & 0.75 \\
\hline
(2, 1)$^-$ & 1.02 & 0.80 & 1.07 \\
\hline
(2, 2)$^+$ & 1.02 & 0.80 & 0.91 \\
\hline
$^-$corner $^+$H-wall edge $^-$E-wall edge $^+$center
\end{tabular}
\end{table}

Due to the symmetry of the problem, $Z_g$ is independent of the index $i$. We then use the impedance matrix $\bar{Z}$ to find all the rms port voltages and currents when one antenna is excited with a unit current and all other antennas are terminated in $Z_g$. This information is used to calculate the scattering parameters $s_{ij}$ for $i, j = 1, \ldots, N$ according to Kurokawa’s classic paper [18]. The coupling to the output TEM waveguide mode is evident from the symmetry of the array: $s_{ii} = s_{i0} = -1/\sqrt{N}$ for $i = 1, \ldots, N$. We also deduce $s_{00}$ is zero.

The full $(N + 1)$-port scattering matrix $\bar{S}$ can be used for a number of applications. For example, our analysis can be used to show that if the $N$ input antennas are excited by uncorrelated noise sources with equal powers, the output noise power will be identical to the noise power available at any one input. The matrix $\bar{S}$ may also be useful for studying locking dynamics and phase noise in oscillator arrays.

In this paper, however, we concentrate on failure analysis. As an example, we will consider a dipole array operating at 10 GHz with an element spacing $u$ of 8 mm. Each dipole is 7.6 mm long and 0.8 mm wide. We assume a triangular surface current distribution on the dipole, maximum in the center and tapering linearly to zero at the ends. Tables I and II show the reduction in output power when a single element fails in a 9 and 25-element array, respectively. We consider various failure locations in the array, and also consider devices failing as a match, open, and short. These quantities are easily calculated using (8) and the scattering parameters. For all of these single-device failures, failure as a short is more detrimental to the output than failure as an open. For matched failures, the loss is independent of the fault’s position in the array.

Fig. 5 plots the simulated cumulative change in output power for a 100-element dipole array in a hard-wall waveguide for multiple failures. Again, we consider device failure as a match, open, and short. The location of the faults is determined randomly. When the devices fail as a match, the reduction in output power follows our archetypal failure expression (10). As before, the loss does not depend on the location of a matched fault. When devices fail as an open or short, however, the reduction in power does depend on device location. For these cases, we plot the average of 100 random trials. All three cases are similar for a relatively few number of failures. As more devices fail, however, the curves separate. As before, the best case is when devices...
fail as open circuits. The worst case is when devices fail as matches. Unlike open or short circuits, the matched failures can absorb power that could otherwise be available to the output. Nevertheless, the grid in a hard-wall waveguide does fail gracefully.

Also plotted in Fig. 5 are measured results from Cheng and York's 2×2 power amplifier array in standard X-band waveguide. This array is similar to the one reported in [13]. Device failure is simulated by turning off the element bias. The measured results are in very good agreement with our archetypal equation (10).

### V. Arrays in Free Space

A similar approach can be used to simulate dipole arrays in free space. In this case, the self and mutual impedances are calculated using the induced EMF method. The calculations are outlined in many antenna textbooks [16, 19, 20] and will not be repeated here. The output port represents power radiated normally from the array’s surface. We compute the scattering matrix $\tilde{S}$ in a similar manner as in the previous section. Each element is terminated with an identical normalizing impedance $Z_0$; this impedance is the conjugate of the driving-point impedance of a central element. One notable difference between free-space and waveguide arrays is in calculating the coupling to the output. We cannot deduce the $s_{ij}$ elements by symmetry, and must compute the radiated wave using the input current at antenna $j$ and the induced currents on all of the other antennas. The $s_{ij}$ will not, in general, be identical. As a result, equations (8)–(10) are no longer valid. Since we compute the entire scattering matrix $\tilde{S}$, however, we can calculate the change in output directly.

Tables III and IV show the reduction in output power when a single element fails in a 9 and 25-element array, respectively. The grid dimensions are the same as those in the previous waveguide section. Again, we consider various failure locations in the array. We also consider devices failing as a match, open, and short. As before, a single shorted failure is the worst case.

Fig. 6 shows the simulated cumulative change in output power for a 100-element free-space array with multiple failures. Even though the scattering parameters of the free-space array are quite different from the waveguide array, the failure results are nearly identical. This suggests that the
This frequency, a single 7-mm strip dipole is resonant. The performance of power-combining arrays as devices begin to fail may be somewhat insensitive to the details of the array construction. We note that although equation (10) is no longer strictly valid, the simulated results for matched failures are indistinguishable from (10). Also plotted in Fig. 6 is Kim’s measured results for a 106-element HBT amplifier array [2]. Kim’s array was tested at 10 GHz and had a unit cell size of 8 mm. Device failures were simulated by detuning the amplifier’s input match. The simulation agrees very well with the measured data for a 10% failure rate. The agreement becomes worse as more devices fail. To some extent this is to be expected, as our simulation does not account for any change to the grid amplifier’s input power as the devices fail. Furthermore, Kim’s array was constructed on a dielectric substrate and included polarizing and tuning elements. Our simple simulation does not include these structures, and assumes a greatly simplified surface current distribution.

We also simulate the cumulative loss for the same free-space array with the frequency increased to 17.9 GHz. At this frequency, a single 7.6-mm strip dipole is resonant. The failure performance is similar to the results at 10 GHz with one notable exception: the performance with shorted failures is worse. This is to be expected as the shorted failures become resonant parasitic elements. For comparison, shorted failures at 17.9 GHz are also shown in Fig. 6.

We finally note that as devices fail in a free-space array, the array’s radiation pattern will change. Our analysis takes these changes into account only to the extent that affects normal radiation. A detailed analysis of the effects of element failure on the radiation pattern including sidelobes can be found in the work of Mailloux and Cohen [21].

VI. Conclusion

We have developed a simple formula for the loss in power combiners with identical, matched sources. To apply the formula, we need to know the reflection coefficients of the fault and the input port on the power combiner. We have applied the formula to two simple grids. We have also outlined a method to determine the scattering parameters of an N-element array with one output port. We apply our method to analyze failures in dipole power-combining arrays in free-space and waveguide. These arrays are shown to degrade gracefully, in agreement with experiment. For a relatively few number of failures all cases presented are very similar; equation (10) can be used as a guide to predict the change in output in power-combiners as elements begin to fail. Our examples have been admittedly simplified to facilitate computation. The approach, however, is general. If necessary, more sophisticated methods could be used to determine the scattering matrix, like the full-wave approach developed by Nuteson, Steer, and co-workers [22, 23].

VIII. References

David Rutledge (S77–M77–SM89–F93) is Professor of Electrical Engineering at the California Institute of Technology. His research has been in developing integrated-circuit antennas, imaging arrays, active grids, and software for computer-aided design and measurement. He is co-author with Scott Wadge and Richard Compton of the widely distributed educational microwave computer-aided design package, Puff, with over 20,000 copies worldwide. Five of his students have won Presidential Investigator and Career Awards. His group has contributed 200 publications to the technical literature.

Prof. Rutledge was Distinguished Lecturer for the Antennas & Propagation Society, won the Microwave Prize of the Microwave Theory & Techniques Society and the Teaching Award of the Associated Students of Caltech, and is a Fellow of the Institute of Electrical and Electronic Engineers.

Nai-Shuo Cheng (S97) received the B.S. degree in nuclear engineering from National Tsing Hua University, Hsinchu, Taiwan, in 1989, and the M.S.E.E. degree in electrical engineering from Syracuse University, Syracuse, NY, in 1994, and is currently working towards the Ph.D. degree in electrical engineering at the University of California at Santa Barbara. His current research involves the development of waveguide-based spatial power combiner circuits and study of the propagation characteristics for non-TEM wave-guiding structure by using finite difference method. He received second place in the Student Paper Competition at the 1998 IEEE International Microwave Symposium.

Robert A. York (S’86–M’91) received the B.S. degree in electrical engineering from the University of New Hampshire in 1987, and the M.S. and Ph.D. degrees in electrical engineering at Cornell University in 1989 and 1991, respectively. He is currently an Associate Professor of Electrical and Computer Engineering at the University of California at Santa Barbara. His group at UCSB is currently involved with the design and fabrication of novel microwave and millimeter-wave circuits, microwave photonics, high power microwave and millimeter-wave modules using spatial combining and wide-bandgap semiconductor devices, and application of ferroelectric materials to millimeter-wave circuits and systems. Dr. York received the Army Research Office Young Investigator Award in 1993, and the Office of Naval Research Young Investigator award in 1996.

Robert M. Weikle II (S’90–M’91) was born in Tacoma, WA, in 1963. He received the B.S. degree in electrical engineering and physics from Rice University, Houston, TX, in 1986 and the M.S. and Ph.D. degrees from the California Institute of Technology, Pasadena, CA, in 1987 and 1992, respectively.

From January to December of 1992, he was a Post-Doctoral Research Scientist in the Department of Applied Electron Physics, Chalmers University, Gothenburg, Sweden. While at Chalmers, he worked on millimeter-wave HEMT mixers and superconducting hot electron bolometers. In 1993, he joined the faculty at the University of Virginia, Charlottesville, where he is now an Assistant Professor in the Department of Electrical Engineering. His current research interests include microwave and millimeter-wave solid state devices, quasi-optical circuits and techniques, and high-frequency receivers and multipliers.

Dr. Weikle is a member of the American Physical Society, International Union of Radio Scientists (Commission D), Phi Beta Kappa, Tau Beta Pi, and Eta Kappa Nu.

Michael P. De Lio (S’86–A’95–M’96) was born in Southfield, Michigan, on July 29, 1968. He received the B.S.E. degree in electrical engineering from the University of Michigan, Ann Arbor, in 1990. In 1991, he obtained the M.S. degree in electrical engineering from the California Institute of Technology in Pasadena. He received the Ph.D. degree in 1996 from Caltech.

In January, 1996 he joined the Department of Electrical Engineering at the University of Hawai‘i at Mānoa as an Assistant Professor. His research interests include high-frequency solid-state devices, microwave and millimeter-wave power combining, and monolithic quasi-optical devices.

Prof. De Lio is a member of Tau Beta Pi, Eta Kappa Nu, and the American Society for Engineering Education. He is the secretary to the IEEE MTT-S Administrative Committee (AdCom) in 1999.