Vector Mechanics for Engineers: Statics

How to prepare for the midterm

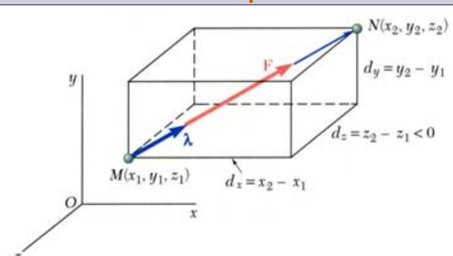
- The midterm will be based on Chapters 1-5 and sections 6.1-6.7. It will be one-hour, take-home, open-textbook and open-notes exam.
- Read "Review and Summary" after each Chapter. Brush up on topics that are not familiar.
- Make sure you know how to solve HW problems and sample problems. It is useful to review all sample problems, or at least 2.9, 3.4, 3.5, 3.7, 4.2, 4.3, 4.4, 4.5, 5.1, 5.2, 5.4, 5.6, 5.9, 5.10, 6.1, 6.2.
- Review important tables/formulae from the book (such as supports and their reactions) so that you can use them easily.
- Remember, the correct reasoning and an error in computation will get you most of the points. However, the right answer with no explanation will get you no points, unless the problem specifically asks for an answer only.
- Do not forget about the honor code. Carefully read the instructions on the front page of the midterm. You cannot discuss anything about the midterm until after the due date.
- The rest of this handout, is a brief summary of important topics we have learned so far.

Vector Mechanics for Engineers: Statics

Force defined by its magnitude and two points

Often the force is defined by its magnitude F and by two points on its line of action,

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = d_x\vec{i} + d_y\vec{j} + d_z\vec{k} = \text{vector joining } M \text{ and } N$$

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1 \quad d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$\vec{F} = F\vec{\lambda} \quad F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Vector Mechanics for Engineers: Statics

Moment of a Force About a Point

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on its line of action.
- The *moment* of $\boldsymbol{F}$ about O is defined as

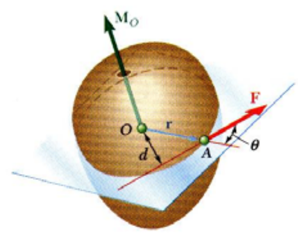
$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

- The moment vector $\boldsymbol{M_O}$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.
- Magnitude of $\boldsymbol{M_O}$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M_O}$.

$$M_O = rF\sin\theta = Fd$$

The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F'}$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



(a)

(b)

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Vector Mechanics for Engineers: Statics

Rectangular Components of the Moment of a Force

The moment of $\boldsymbol{F}$ about O,

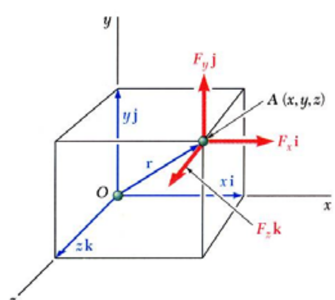
$$\vec{M}_O = \vec{r} \times \vec{F}, \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\vec{i} + (zF_x - xF_z)\vec{j} + (xF_y - yF_x)\vec{k}$$

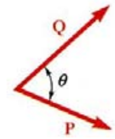


Scalar Product of Two Vectors: Applications

- Angle between two vectors:

$$\vec{P} \bullet \vec{Q} = PQ\cos\theta = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ}$$

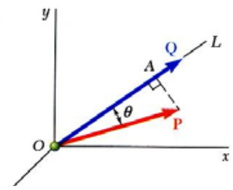


- Projection of a vector on a given axis:

$$P_{OL} = P\cos\theta = \text{projection of } P \text{ along } OL$$

$$\vec{P} \bullet \vec{Q} = PQ\cos\theta$$

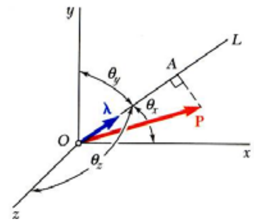
$$\frac{\vec{P} \bullet \vec{Q}}{Q} = P\cos\theta = P_{OL}$$



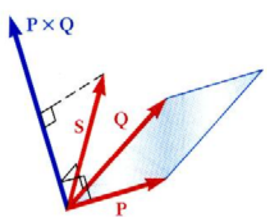
- For an axis defined by a unit vector:

$$P_{OL} = \vec{P} \bullet \vec{\lambda}$$

$$= P_x \cos\theta_x + P_y \cos\theta_y + P_z \cos\theta_z$$



Mixed Triple Product of Three Vectors



- Mixed triple product of three vectors,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \text{scalar result}$$

- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \vec{P} \bullet \left(\vec{Q} \times \vec{S}\right) = \vec{Q} \bullet \left(\vec{S} \times \vec{P}\right)$$

$$= -\vec{S} \bullet \left(\vec{Q} \times P\right) = -\vec{P} \bullet \left(\vec{S} \times \vec{Q}\right) = -\vec{Q} \bullet \left(\vec{P} \times \vec{S}\right)$$

- Evaluating the mixed triple product,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = S_x\left(P_y Q_z - P_z Q_y\right) + S_y\left(P_z Q_x - P_x Q_z\right) + S_z\left(P_x Q_y - P_y Q_x\right)$$

$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment of a Force About a Given Axis

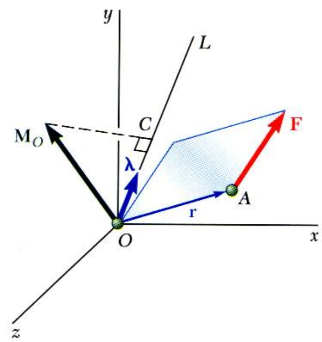
- Moment $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point $\boldsymbol{A}$ about a point $\boldsymbol{O}$,

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- Scalar moment M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

$$M_{OL} = \vec{\lambda} \bullet \vec{M}_O = \vec{\lambda} \bullet \left(\vec{r} \times \vec{F}\right)$$

- The moment M_{OL} of $\boldsymbol{F}$ about the axis OL measures the tendency of the force $\boldsymbol{F}$ to impart a rigid body rotation about the axis OL.



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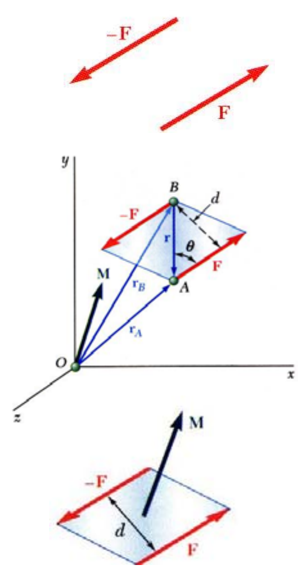
Moment of a Couple

- Two forces $\boldsymbol{F}$ and $\boldsymbol{-F}$ having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.

- Moment of the couple,

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF \sin\theta = Fd$$

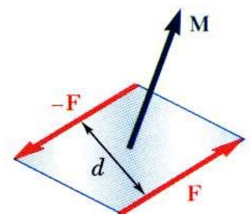
- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

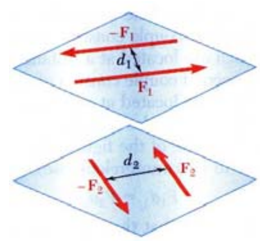


Moment of a Couple

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.



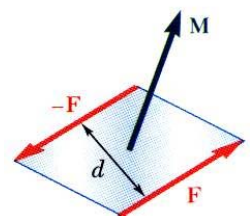


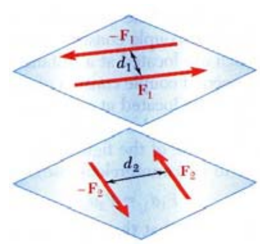
Equivalent Couples

Two systems of forces are equivalent if we can transform one of them into the other by one or more of the following operations:

- replacing two forces acting on the same particle by their resultant;
- resolving the force into components (including attaching two equal and opposite forces to the same particle);
- moving the force along its line of action.

Two couples that have the same moments are equivalent.





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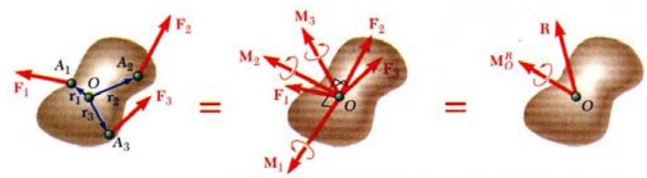
System of Forces: Reduction to a Force and Couple



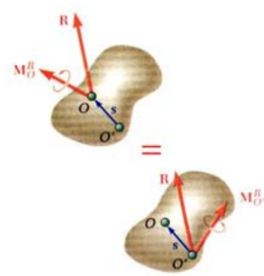
- A system of forces may be replaced by a collection of force-couple systems acting a given point *O*
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum\left(\vec{r} \times \vec{F}\right)$$

- The force-couple system at *O* may be moved to *O'* with the addition of the moment of ***R*** about *O'* ,

$$\vec{M}_{O'}^R = \vec{M}_O^R + \vec{s} \times \vec{R}$$

- Two systems of forces are equivalent if they can be reduced to the same force-couple system.



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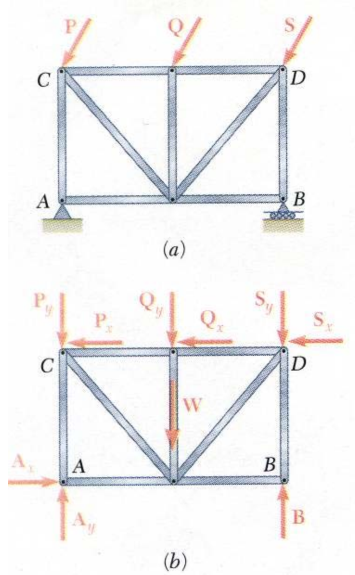
Equilibrium of a Rigid Body in Two Dimensions



- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

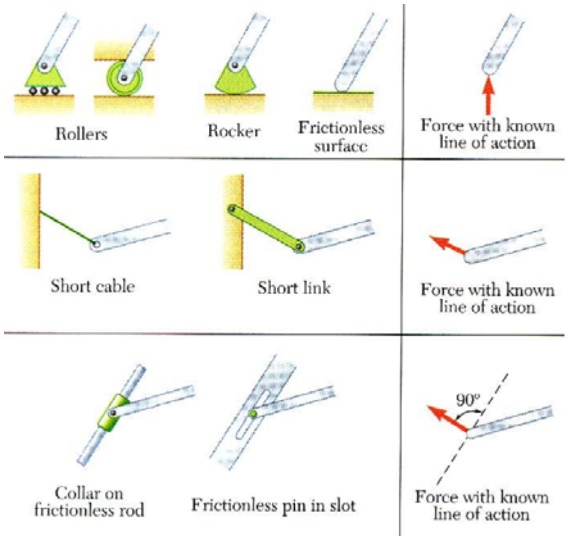
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

where *A* is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.
- The 3 equations can not be augmented with additional equations, but they can be replaced

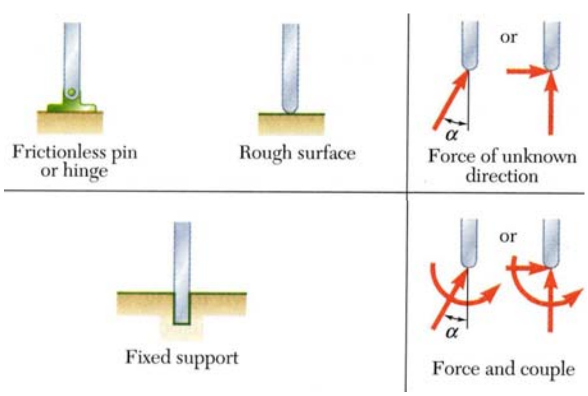
$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Reactions at Supports and Connections for a Two-Dimensional Structure



- Reactions equivalent to a force with known line of action.

Reactions at Supports and Connections for a Two-Dimensional Structure



- Reactions equivalent to a force of unknown direction and magnitude.
- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.
- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum\left(\vec{r} \times \vec{F}\right) = 0$$

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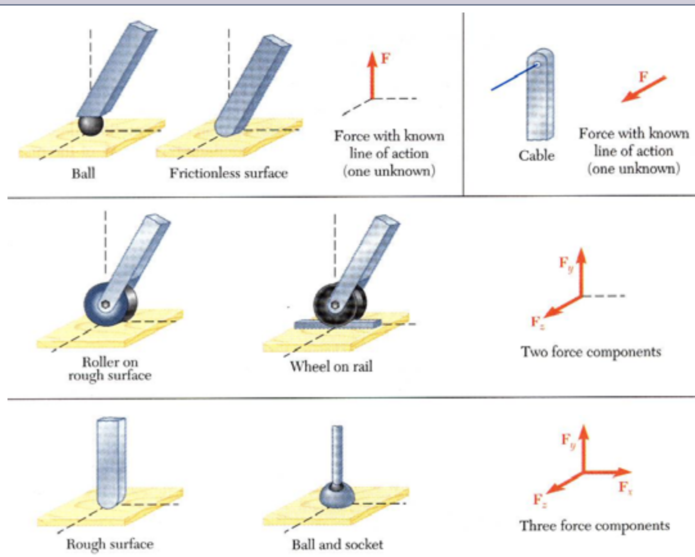
Reactions at Supports and Connections for a Three-Dimensional Structure



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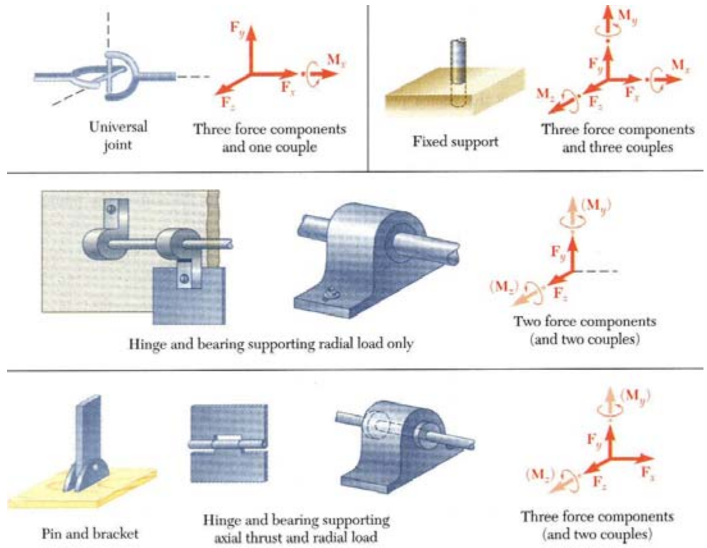
Reactions at Supports and Connections for a Three-Dimensional Structure



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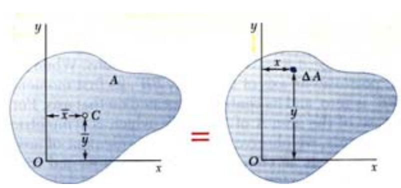
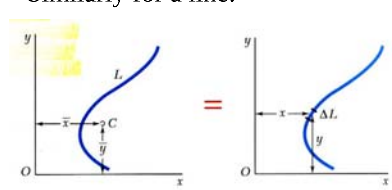
Vector Mechanics for Engineers: Statics

Centroids and First Moments of Areas and Lines



Similarly for a line:



First moments of the area *A*:

$$Q_y = \int x\,dA, \quad Q_x = \int y\,dA$$

Coordinates $\bar{x}, \bar{y}$ of the centroid of the area *A*:

$$\bar{x}A = \int x\,dA = Q_y, \quad \bar{y}A = \int y\,dA = Q_x$$

$$\bar{x}W = \int x\,dW$$

$$\bar{x}(\gamma a L) = \int x(\gamma a)\,dL$$

$$\bar{x}L = \int x\,dL$$

$$\bar{y}L = \int y\,dL$$

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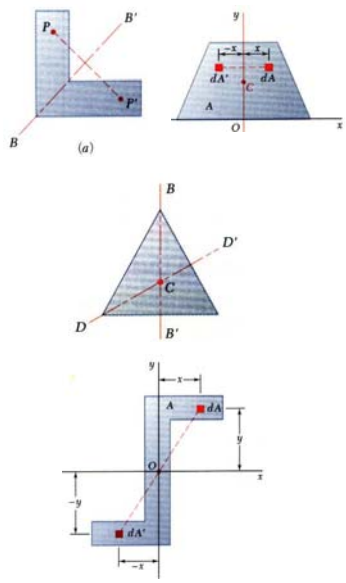
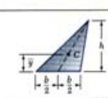
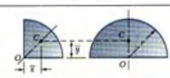
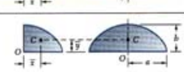
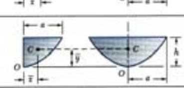
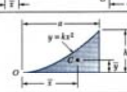
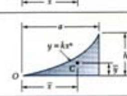
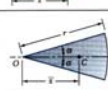
First Moments of Areas and Lines



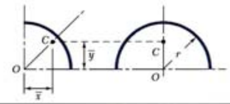
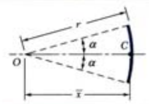
- An area is symmetric with respect to an axis *BB'* if for every point *P* there exists a point *P'* such that *PP'* is perpendicular to *BB'* and is divided into two equal parts by *BB'*.
- The first moment of an area with respect to a line of symmetry is zero.
- If an area possesses a line of symmetry, its centroid lies on that axis
- If an area possesses two lines of symmetry, its centroid lies at their intersection.
- An area is symmetric with respect to a center *O* if for every element *dA* at (*x*,*y*) there exists an area *dA'* of equal area at (-*x*,-*y*).
- The centroid of the area coincides with the center of symmetry.

Vector Mechanics for Engineers: Statics

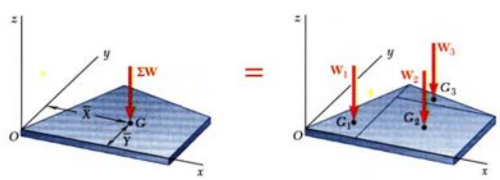
Centroids of Common Shapes of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

Centroids of Common Shapes of Lines

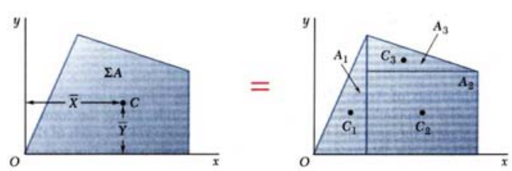
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Composite Plates and Areas



- Composite plates

$$\bar{X}\sum W = \sum \bar{x} W$$
$$\bar{Y}\sum W = \sum \bar{y} W$$



- Composite area

$$\bar{X}\sum A = \sum \bar{x} A$$
$$\bar{Y}\sum A = \sum \bar{y} A$$

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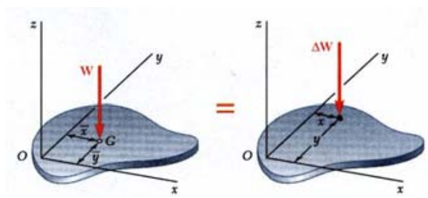
Center of Gravity of a 2D Body

- Center of gravity of a plate



$$\sum F_z: \quad \sum \Delta W, \quad W = \int dW$$

$$\sum M_y: \quad \sum x \Delta W, \quad \bar{x} W = \int x \, dW$$

$$\sum M_y: \quad \sum y \Delta W, \quad \bar{y} W = \int y \, dW$$

Plate of uniform thickness t, specific weight γ, and area A:

$W = \gamma t A$;

$\bar{x}(\gamma t A) = \int x(\gamma t) \, dA$;

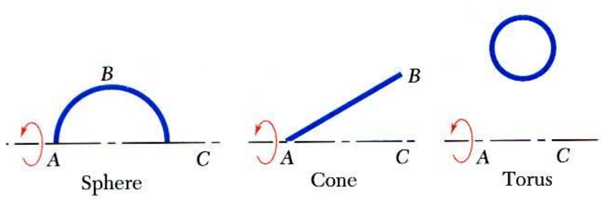
$\bar{x} A = \int x \, dA = Q_y$

= first moment of the area A with respect to the y axis

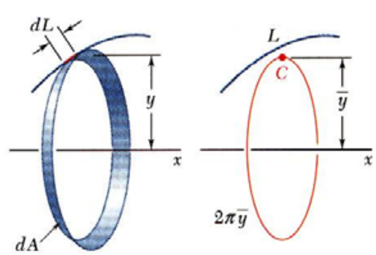
$\bar{y} A = \int y \, dA = Q_x$

Vector Mechanics for Engineers: Statics

Theorems of Pappus-Guldinus



- Surface of revolution is generated by rotating a plane curve about a fixed axis.

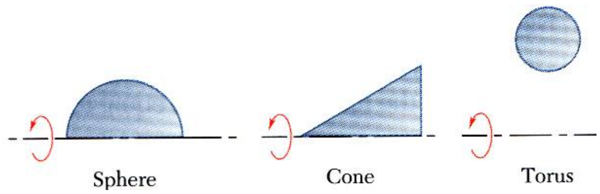


- Area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid through the rotation.

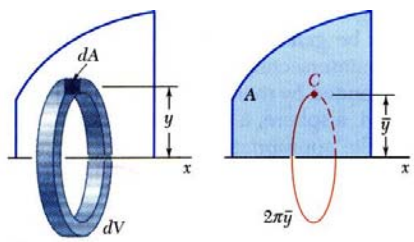
$$A = 2\pi \bar{y} L$$

Vector Mechanics for Engineers: Statics

Theorems of Pappus-Guldinus



- Body of revolution is generated by rotating a plane area about a fixed axis.

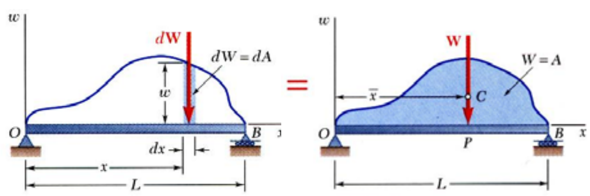


- Volume of a body of revolution is equal to the generating area times the distance traveled by the centroid through the rotation.

$$V = 2\pi \bar{y} A$$

Vector Mechanics for Engineers: Statics

Distributed Loads on Beams



$$W = \int_0^L w\,dx = \int dA = A$$

- A distributed load is represented by plotting the load per unit length, *w* (N/m) . The total load is equal to the area under the load curve.

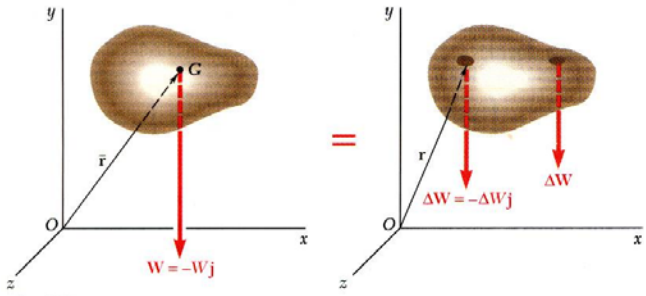
$$(OP)W = \int x\,dW$$

$$(OP)A = \int_0^L x\,dA = \bar{x}A$$

- A distributed load can be replace by a concentrated load with a magnitude equal to the area under the load curve and a line of action passing through the area centroid.

Vector Mechanics for Engineers: Statics

Center of Gravity of a 3D Body: Centroid of a Volume



- Center of gravity G

$$-W\vec{j} = \sum\left(-\Delta W\vec{j}\right)$$

$$\vec{r}_G \times \left(-W\vec{j}\right) = \sum\left[\vec{r} \times \left(-\Delta W\vec{j}\right)\right]$$
$$\vec{r}_G W \times \left(-\vec{j}\right) = \left(\sum \vec{r}\Delta W\right) \times \left(-\vec{j}\right)$$

$$W = \int dW \qquad \vec{r}_G W = \int \vec{r}\, dW$$

- Results are independent of body orientation,

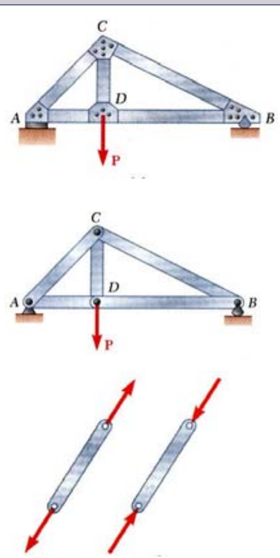
$$\bar{x}W = \int x\,dW \quad \bar{y}W = \int y\,dW \quad \bar{z}W = \int z\,dW$$

- For homogeneous bodies,

$$W = \gamma V \text{ and } dW = \gamma\, dV$$
$$\bar{x}V = \int x\,dV \quad \bar{y}V = \int y\,dV \quad \bar{z}V = \int z\,dV$$

Vector Mechanics for Engineers: Statics

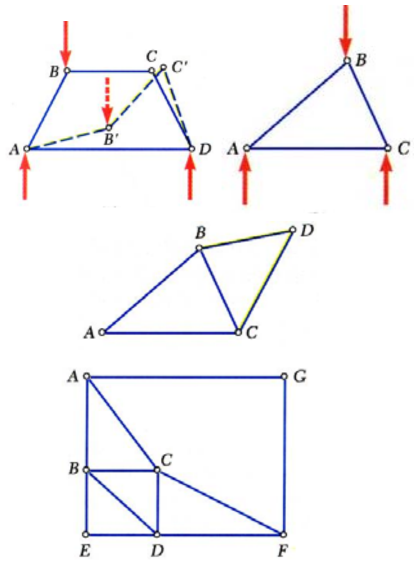
Definition of a Truss



- A truss consists of straight members connected at joints. No member is continuous through a joint.
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Vector Mechanics for Engineers: Statics

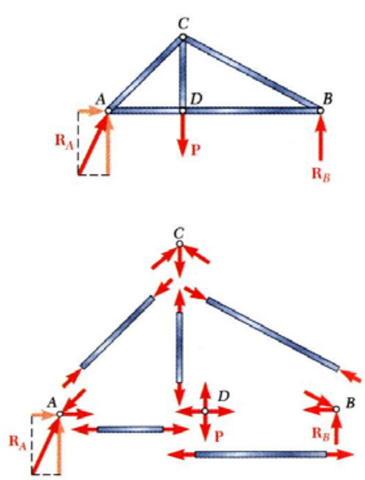
Simple Trusses



- A *rigid truss* will not collapse under the application of a load.
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.
- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

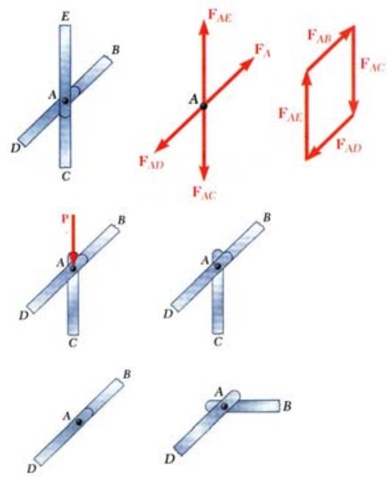
Vector Mechanics for Engineers: Statics

Analysis of Trusses by the Method of Joints

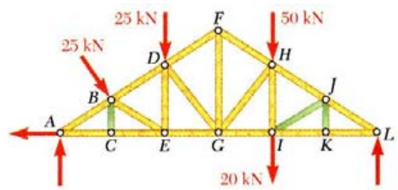


- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

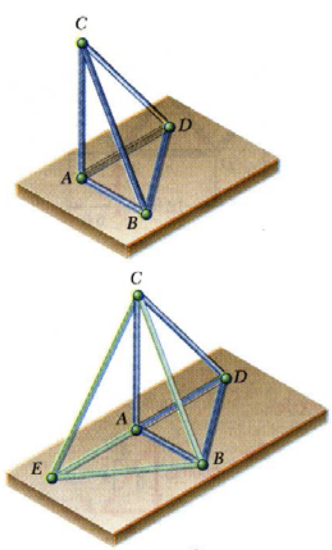
Joints Under Special Loading Conditions



- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.

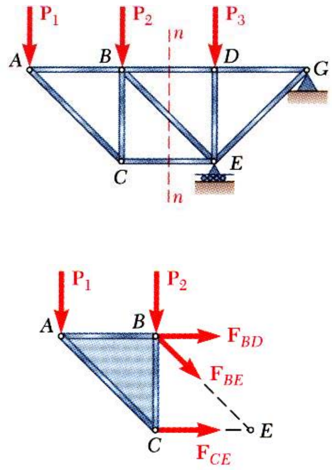


Space Trusses



- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

Analysis of Trusses by the Method of Sections



- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD, pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.