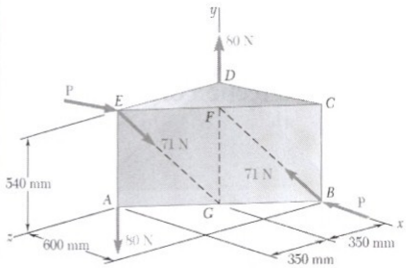
PROBLEM 3.74



Knowing that $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Have $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

Where $\mathbf{M}_1 = \mathbf{r}_{E/D} \times \mathbf{F}_D$

$$= -(0.7 \text{ m})\mathbf{k} \times (80 \text{ N})\mathbf{j}$$

$$= (56.0 \text{ N}\cdot\text{m})\mathbf{i}$$

And $\mathbf{M}_2 = \mathbf{r}_{G/F} \times \mathbf{F}_B$

Now $d_{BF} = \sqrt{(-0.300 \text{ m})^2 + (0.540 \text{ m})^2 + (0.350 \text{ m})^2}$

$$= 0.710 \text{ m}$$

Then $\mathbf{F}_B = \lambda_{BF} F_B$

$$= \frac{(-0.300 \text{ m})\mathbf{i} + (0.540 \text{ m})\mathbf{j} + (0.350 \text{ m})\mathbf{k}}{0.710 \text{ m}}(71 \text{ N})$$

$$= -(30 \text{ N})\mathbf{i} + (54 \text{ N})\mathbf{j} + (35 \text{ N})\mathbf{k}$$

$\therefore$ $\mathbf{M}_2 = (0.54 \text{ m})\mathbf{j} \times \left[-(30 \text{ N})\mathbf{i} + (54 \text{ N})\mathbf{j} + (35 \text{ N})\mathbf{k}\right]$

$$= (18.90 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}$$

Finally $\mathbf{M} = (56.0 \text{ N}\cdot\text{m})\mathbf{i} + \left[(18.90 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}\right]$

$$= (74.9 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{k}$$

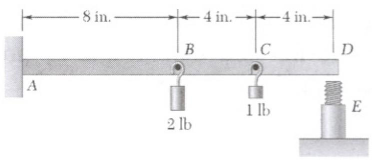
and $M = \sqrt{(74.9 \text{ N}\cdot\text{m})^2 + (16.20 \text{ N}\cdot\text{m})^2}$

$$= 76.632 \text{ N}\cdot\text{m}$$

or $\mathbf{M} = 76.6 \text{ N}\cdot\text{m}$ ◀

$$\cos\theta_x = \frac{74.9}{76.632} \qquad \cos\theta_y = \frac{0}{76.632} \qquad \cos\theta_z = \frac{16.20}{76.632}$$

or $\theta_x = 12.20°$ $\quad\theta_y = 90.0°$ $\quad\theta_z = 77.8°$ ◀

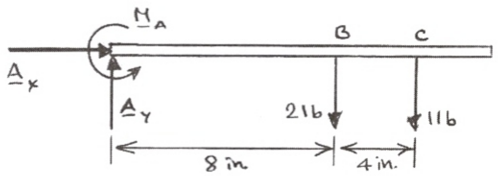


PROBLEM 4.48

In a laboratory experiment, students hang the weights shown from a beam of negligible weight. (*a*) Determine the reaction at the fixed support *A* knowing that end *D* of the beam does not touch support *E*. (*b*) Determine the reaction at the fixed support *A* knowing that the adjustable support *E* exerts an upward force of 1.2 lb on the beam.

SOLUTION

(*a*) **Free-Body Diagram:**



$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0$: $\quad A_y - 2\text{ lb} - 1\text{ lb} = 0$

$$A_y = 3\text{ lb}$$

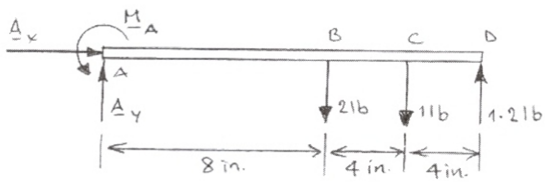
or $\mathbf{A} = 3\text{ lb}\uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0$: $\quad M_A - (2\text{ lb})(8\text{ in.}) - (1\text{ lb})(12\text{ in.}) = 0$

$$M_A = 28\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_A = 28\text{ lb}\cdot\text{in.} \circlearrowleft$ ◀

(*b*) **Free Body Diagram:**



$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0$: $\quad A_y - 2\text{ lb} - 1\text{ lb} + 1.2\text{ lb} = 0$

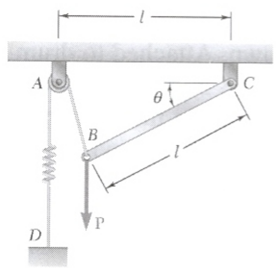
$$A_y = 1.8\text{ lb}$$

or $\mathbf{A} = 1.8\text{ lb}\uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0$: $\quad M_A - (2\text{ lb})(8\text{ in.}) - (1\text{ lb})(12\text{ in.}) + (1.2\text{ lb})(16\text{ in.}) = 0$

$$M_A = 8.8\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_A = 8.8\text{ lb}\cdot\text{in.} \circlearrowleft$ ◀

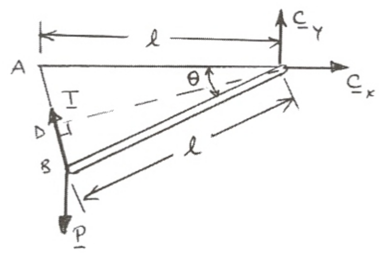


PROBLEM 4.59

A vertical load **P** is applied at end B of rod BC. The constant of the spring is k and the spring is unstretched when $\theta = 0$. (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P, k, and l. (b) Determine the value of θ corresponding to equilibrium if $P = 2kl$.

SOLUTION

Free-Body Diagram:



Geometry:

Triangle ABC is isosceles. Thus distance $CD = l\cos\dfrac{\theta}{2}$,

Elongation of spring is equal to distance AB:

$$x = 2l\sin\frac{\theta}{2},$$

and $\quad T = kx = 2kl\sin\dfrac{\theta}{2}.$

(a) Equilibrium for rod:

$$+\circlearrowleft\ \Sigma M_C = 0: \qquad P(l\cos\theta) - T\left(l\cos\frac{\theta}{2}\right) = 0$$

$$Pl\cos\theta - kl^2\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = 0$$

$$P\cos\theta - kl\sin\theta = 0$$

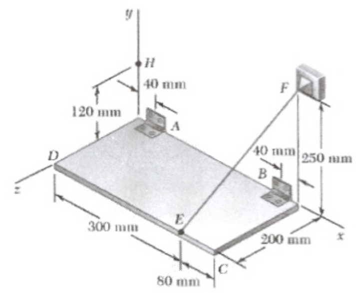
$$\tan\theta = \frac{P}{kl}$$

or $\theta = \tan^{-1}\left(\dfrac{P}{kl}\right)$ ◀

(b) For $p = 2kl$:

$$\theta = \tan^{-1}\left(\frac{2kl}{kl}\right) = \tan^{-1}(2) = 63.435°$$

or $\theta = 63.4°$ ◀

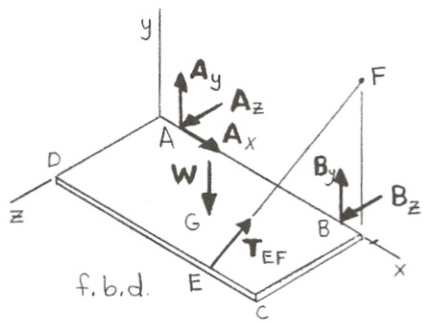


PROBLEM 4.122

The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges A and B and cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



First note $$W = mg = (15 \text{ kg})(9.81 \text{ m/s}^2) = 147.15 \text{ N}$$

(*a*)

$$\mathbf{T}_{EF} = \boldsymbol{\lambda}_{EF} T_{EF} = \left[\frac{(0.08 \text{ m})\mathbf{i} + (0.25 \text{ m})\mathbf{j} - (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.08)^2 + (0.25)^2 + (0.2)^2} \text{ m}}\right] T_{EF} = \frac{T_{EF}}{0.33}(0.08\mathbf{i} + 0.25\mathbf{j} - 0.2\mathbf{k})$$

From free-body diagram of rectangular plate

$$\Sigma M_x = 0: \quad (147.15 \text{ N})(0.1 \text{ m}) - (T_{EF})_y (0.2 \text{ m}) = 0$$

or $$14.715 \text{ N}\cdot\text{m} - \left[\left(\frac{0.25}{0.33}\right) T_{EF}\right](0.2 \text{ m}) = 0$$

or $$T_{EF} = 97.119 \text{ N}$$

or $T_{EF} = 97.1 \text{ N}$ ◀

(*b*) $$\Sigma F_x = 0: \quad A_x + (T_{EF})_x = 0$$

$$A_x + \left(\frac{0.08}{0.33}\right)(97.119 \text{ N}) = 0$$

$$\therefore \quad A_x = -23.544 \text{ N}$$

PROBLEM 4.122 CONTINUED

$\Sigma M_{B(z\text{-axis})} = 0$: $\quad -A_y(0.3\text{ m}) - (T_{EF})_y(0.04\text{ m}) + W(0.15\text{ m}) = 0$

or

$$-A_y(0.3\text{ m}) - \left[\left(\frac{0.25}{0.33}\right)97.119\text{ N}\right](0.04\text{ m}) + 147.15\text{ N}(0.15\text{ m}) = 0$$

$$\therefore \quad A_y = 63.765\text{ N}$$

$\Sigma M_{B(y\text{-axis})} = 0$: $\quad A_z(0.3\text{ m}) + (T_{EF})_x(0.2\text{ m}) + (T_{EF})_z(0.04\text{ m}) = 0$

$$A_z(0.3\text{ m}) + \left[\left(\frac{0.08}{0.33}\right)T_{EF}\right](0.2\text{ m}) - \left[\left(\frac{0.2}{0.33}\right)T_{EF}\right](0.04\text{ m}) = 0$$

$$\therefore \quad A_z = -7.848\text{ N}$$

and $\mathbf{A} = -(23.5\text{ N})\mathbf{i} + (63.8\text{ N})\mathbf{j} - (7.85\text{ N})\mathbf{k}$ ◀

$\Sigma F_y = 0$: $\quad A_y - W + (T_{EF})_y + B_y = 0$

$$63.765\text{ N} - 147.15\text{ N} + \left(\frac{0.25}{0.33}\right)(97.119\text{ N}) + B_y = 0$$

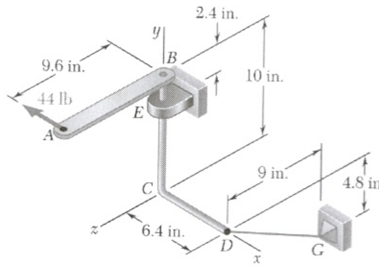
$$\therefore \quad B_y = 9.81\text{ N}$$

$\Sigma F_z = 0$: $\quad A_z - (T_{EF})_z + B_z = 0$

$$-7.848\text{ N} - \left(\frac{0.2}{0.33}\right)(97.119\text{ N}) + B_z = 0$$

$$\therefore \quad B_z = 66.708\text{ N}$$

and $\mathbf{B} = (9.81\text{ N})\mathbf{j} + (66.7\text{ N})\mathbf{k}$ ◀

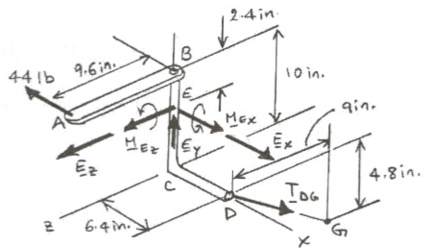


PROBLEM 4.129

The lever *AB* is welded to the bent rod *BCD* which is supported by bearing *E* and by cable *DG*. Assuming that the bearing can exert an axial thrust and couples about axes parallel to the *x* and *z* axes, determine (*a*) the tension in cable *DG*, (*b*) the reaction at *E*.

SOLUTION

Free-Body Diagram:



Express the tension in terms of its rectangular components:

$$\overrightarrow{DG} = -(4.8 \text{ in.})\mathbf{j} - (9 \text{ in.})\mathbf{k}$$

$$\mathbf{T}_{DC} = T_{DG}\frac{\overrightarrow{DG}}{DG} = T_{DG}\frac{-4.8\mathbf{j} - 9\mathbf{k}}{\sqrt{(-4.8)^2 + (-9)^2}} = -\frac{8}{17}T_{DG}\,\mathbf{j} - \frac{15}{17}T_{DG}\,\mathbf{k}$$

$$\Sigma\mathbf{M}_E = 0: \quad \mathbf{M}_E + \left[(6.4 \text{ in.})\mathbf{i} + (-7.6 \text{ in.})\mathbf{j}\right] \times \mathbf{T}_{DG}$$

$$+\left[(2.4 \text{ in.})\mathbf{j} + (9.6 \text{ in.})\mathbf{k}\right] \times (-44 \text{ lb})\mathbf{i} = 0$$

or $$\left(M_{Ex}\,\mathbf{i} + M_{Ey}\,\mathbf{j} + M_{Ez}\,\mathbf{k}\right) + (7.6 \text{ in.})\left(\frac{15}{17}\right)T_{DG}\mathbf{i} - (6.4 \text{ in.})\left(\frac{8}{17}\right)T_{DG}\mathbf{k} + (6.4 \text{ in.})\left(\frac{15}{17}\right)T_{DG}\mathbf{j}$$

$$+(2.4 \text{ in.})(44 \text{ lb})\mathbf{k} - (9.6 \text{ in.})(44 \text{ lb})\mathbf{j} = 0$$

Setting the coefficient of the unit vector **j** equal to zero:

(*a*) **j**: $$\left(\frac{15}{17}T_{DG}\right)(6.4 \text{ in.}) - (44 \text{ lb})(9.6 \text{ in.}) = 0$$

$$T_{DG} = 74.800 \text{ lb}$$

or $T_{DG} = 74.8 \text{ lb}$ ◀

PROBLEM 4.129 CONTINUED

(*b*) $\Sigma F_x = 0$: $\quad E_x - 44\text{ lb} = 0, \text{ or } E_x = 44.000\text{ lb}$

$\Sigma F_y = 0$: $\quad E_y - \dfrac{8}{17}(74.8\text{ lb}) = 0, \text{ or } E_y = 35.200\text{ lb}$

$\Sigma F_z = 0$: $\quad E_z - \dfrac{15}{17}(74.8\text{ lb}) = 0, \text{ or } E_z = 66.000\text{ lb}$

or $\mathbf{E} = (44.0\text{ lb})\mathbf{i} + (35.2\text{ lb})\mathbf{j} + (66.0\text{ lb})\mathbf{k}$ ◀

Using the moment equation again and setting the coefficients of the unit vectors **i** and **k** equal to zero:

i: $\quad M_{Ex} + (7.6\text{ in.})\left(\dfrac{15}{17}\right)(74.800\text{ lb}) = 0$

$$M_{Ex} = -501.60\text{ lb}\cdot\text{in.}$$

k: $\quad M_{Ez} + (44\text{ lb})(2.4\text{ in.}) - \left[\dfrac{8}{17}(74.8\text{ lb})\right](6.4\text{ in.}) = 0$

$$M_{Ex} = 119.680\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_E = -(502\text{ lb}\cdot\text{in.})\mathbf{i} + (119.7\text{ lb}\cdot\text{in.})\mathbf{k}$ ◀