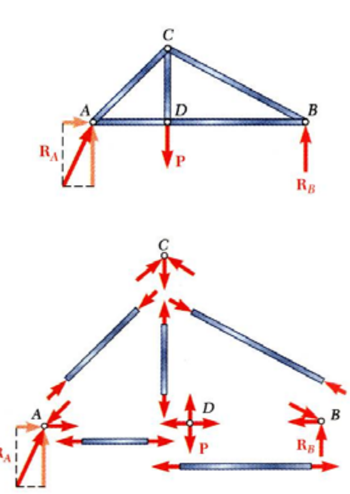
Vector Mechanics for Engineers: Statics

How to prepare for the final

- The final will be based on Chapters 6, 7, 8, and sections 10.1-10.5. It will be three-hour, take-home, open-textbook and open-notes exam.
- Read "Review and Summary" after each Chapter. Brush up on topics that are not familiar.
- Make sure you know how to solve HW problems and sample problems. Additional review problems for the final will be posted on the web.
- Review important tables/formulae from the book (such as supports and their reactions) so that you can use them easily.
- Remember, the correct reasoning and an error in computation will get you most of the points. However, the right answer with no explanation will get you no points, unless the problem specifically asks for an answer only.
- Do not forget about the honor code. Carefully read the instructions on the front page of the final. You cannot discuss anything about the final until after the due date.
- The rest of this document is a brief summary of important topics we have learned in the second half of the term.

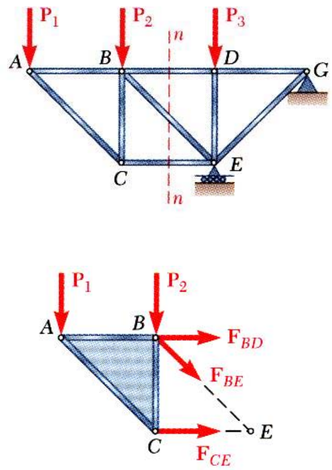
Vector Mechanics for Engineers: Statics

Analysis of Trusses by the Method of Joints



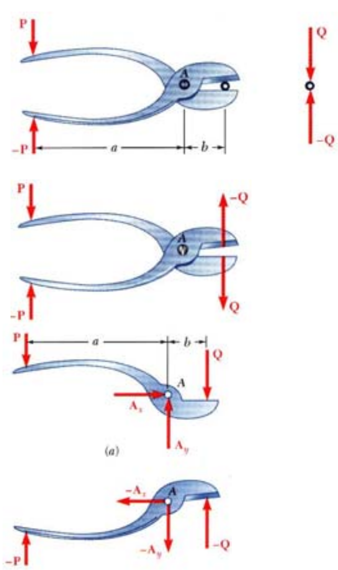
- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide 2*n* equations for 2*n* unknowns. For a simple truss, $2n = m + 3$. May solve for *m* member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

Analysis of Trusses by the Method of Sections



- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD*, *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

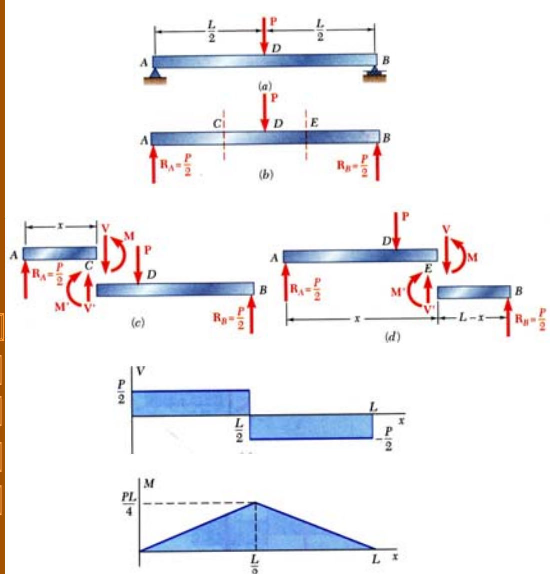
Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Given the magnitude of ***P***, determine the magnitude of ***Q***.
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a nonrigid structure. Use one of the components as a free-body.
- Taking moments about *A*,

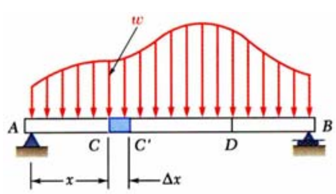
$$\sum M_A = 0 = aP - bQ \qquad Q = \frac{a}{b}P$$

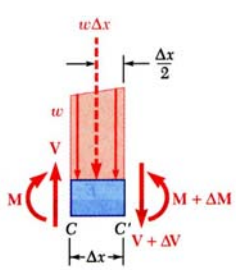
Shear and Bending Moment Diagrams



- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,
  $V = +P/2 \quad M = +Px/2$
- Cut beam at *E* and consider member *EB*,
  $V = -P/2 \quad M = +P(L-x)/2$
- For a beam subjected to concentrated loads, shear is constant between loading points and moment varies linearly.

Relations Among Load, Shear, and Bending Moment





- Relations between load and shear:

$$V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx = -(\text{area under load curve})$$

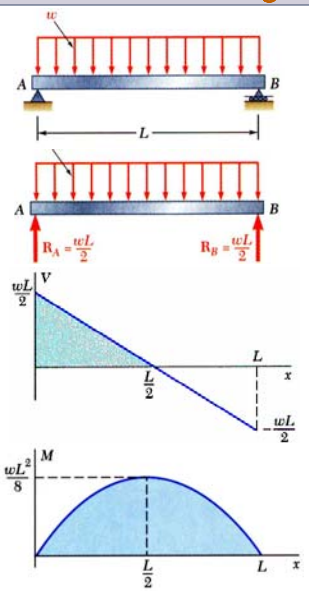
- Relations between shear and bending moment:

$$(M + \Delta M) - M - V\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\frac{dM}{dx} = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \lim_{\Delta x \to 0} \left(V - \tfrac{1}{2} w\Delta x\right) = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx = (\text{area under shear curve})$$

Relations Among Load, Shear, and Bending Moment



- Reactions at supports, $R_A = R_B = \dfrac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

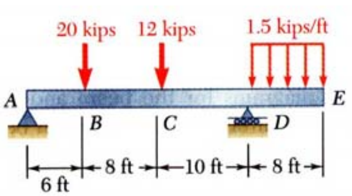
- Moment curve,

$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Sample Problem 7.4

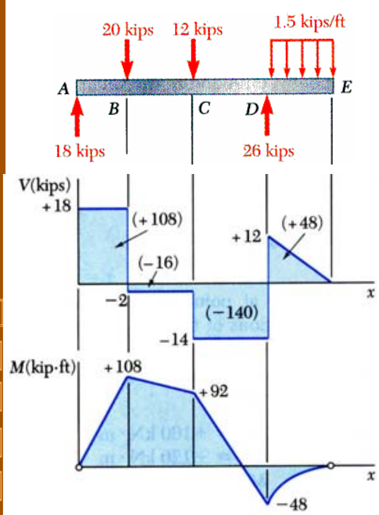


Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.
- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between *D* and *E*, the shear variation is linear.
- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under shear curve between points.
- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

Sample Problem 7.4

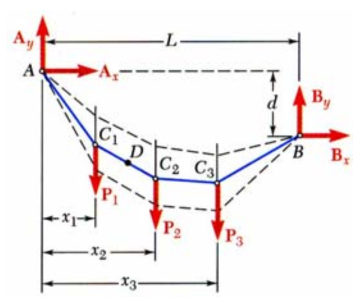


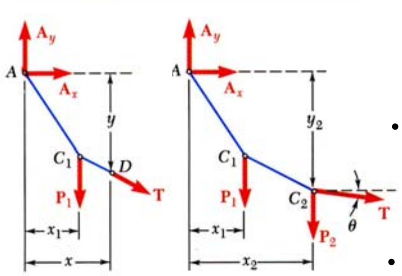
- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under the shear curve between points.

$$M_B - M_A = +108 \qquad M_B = +108\ \text{kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\ \text{kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\ \text{kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

Cables With Concentrated Loads



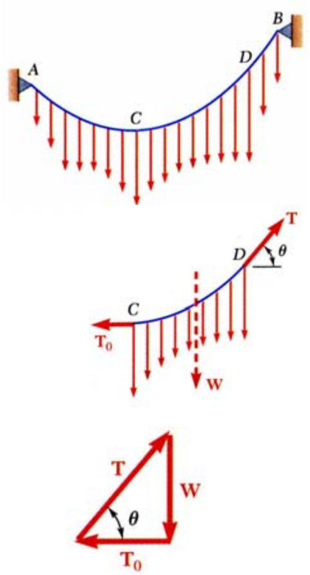


- Consider entire cable as free-body. Slopes of cable at *A* and *B* are not known - two reaction components required at each support.
- Four unknowns are involved and three equations of equilibrium are not sufficient to determine the reactions.
- Additional equation is obtained by considering equilibrium of portion of cable *AD* and assuming that coordinates of point *D* on the cable are known. The additional equation is $\sum M_D = 0$.
- For other points on cable,

  $\sum M_{C_2} = 0$ yields y_2

  $\sum F_x = 0, \sum F_y = 0$ yield T_x, T_y
- $T_x = T\cos\theta = A_x = \text{constant}$

Cables With Distributed Loads



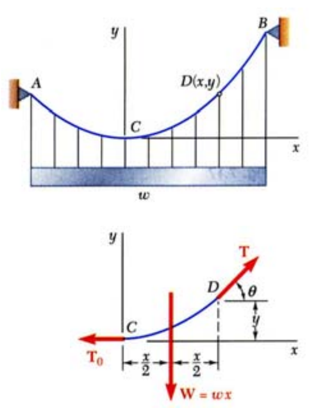
- For cable carrying a distributed load:
  a) cable hangs in shape of a curve
  b) internal force is a tension force directed along tangent to curve.
- Consider free-body for portion of cable extending from lowest point *C* to given point *D*. Forces are horizontal force T_0 at C and tangential force $\boldsymbol{T}$ at *D*.
- From force triangle:

$$T\cos\theta = T_0 \qquad T\sin\theta = W$$

$$T = \sqrt{T_0^2 + W^2} \qquad \tan\theta = \frac{W}{T_0}$$

- Horizontal component of $\boldsymbol{T}$ is uniform over cable.
- Vertical component of $\boldsymbol{T}$ is equal to magnitude of *W* measured from lowest point.
- Tension is minimum at lowest point and maximum at *A* and *B*.

Parabolic Cable



- Consider a cable supporting a uniform, horizontally distributed load, e.g., support cables for a suspension bridge.
- With loading on cable from lowest point *C* to a point *D* given by $W = wx$, internal tension force magnitude and direction are

$$T = \sqrt{T_0^2 + w^2x^2} \qquad \tan\theta = \frac{wx}{T_0}$$

- Summing moments about *D*,

$$\sum M_D = 0: \qquad wx\frac{x}{2} - T_0 y = 0$$

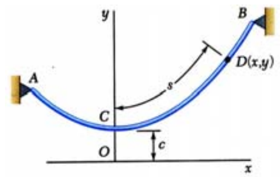
or

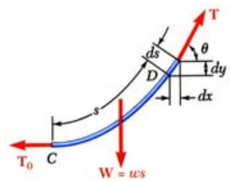
$$y = \frac{wx^2}{2T_0}$$

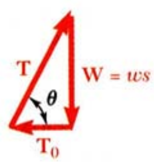
The cable forms a parabolic curve.

Vector Mechanics for Engineers: Statics

Catenary







- Consider a cable uniformly loaded along the cable itself, e.g., cables hanging under their own weight.
- With loading on the cable from lowest point C to a point D given by $W = ws$, the internal tension force magnitude is

$$T = \sqrt{T_0^2 + w^2 s^2} = w\sqrt{c^2 + s^2} \qquad c = T_0 / w$$

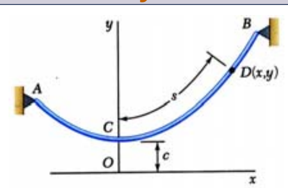
- To relate horizontal distance x to cable length s,

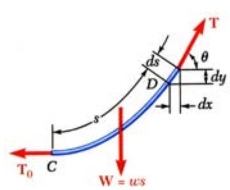
$$dx = ds\cos\theta = \frac{T_0}{T}\cos\theta = \frac{ds}{\sqrt{q + s^2/c^2}}$$

$$x = \int_0^s \frac{ds}{\sqrt{q + s^2/c^2}} = c\sinh^{-1}\frac{s}{c} \quad \text{and} \quad s = c\sinh\frac{x}{c}$$

Vector Mechanics for Engineers: Statics

Catenary





- To relate x and y cable coordinates,

$$dy = dx\tan\theta = \frac{W}{T_0}dx = \frac{s}{c}dx = \sinh\frac{x}{c}\,dx$$

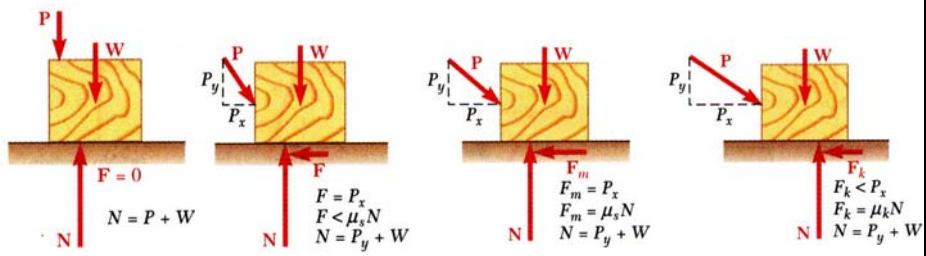
$$y - c = \int_0^x \sinh\frac{x}{c}\,dx = c\cosh\frac{x}{c} - c$$

$$y = c\cosh\frac{x}{c}$$

which is the equation of a catenary.

The Laws of Dry Friction. Coefficients of Friction

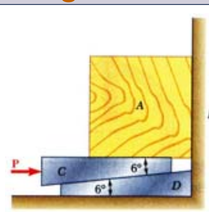
- Four situations can occur when a rigid body is in contact with a horizontal surface:

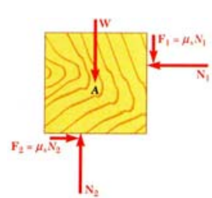


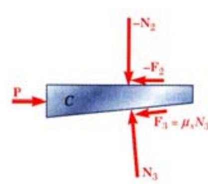
- No friction, ($P_x = 0$)
- No motion, ($P_x < F_m$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

Vector Mechanics for Engineers: Statics
Eighth edition

Wedges







- *Wedges* - simple machines used to raise heavy loads.
- Friction prevents wedge from sliding out.
- Want to find minimum force P to raise block.

- Block as free-body

$$\sum F_x = 0:$$
$$-N_1 + \mu_s N_2 = 0$$
$$\sum F_y = 0:$$
$$-W - \mu_s N_1 + N_2 = 0$$

or

$$\vec{R}_1 + \vec{R}_2 + \vec{W} = 0$$

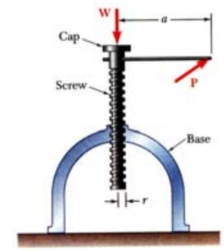
- Wedge as free-body

$$\sum F_x = 0:$$
$$-\mu_s N_2 - N_3(\mu_s \cos 6° - \sin 6°) + P = 0$$
$$\sum F_y = 0:$$
$$-N_2 + N_3(\cos 6° - \mu_s \sin 6°) = 0$$

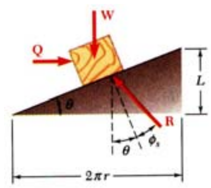
or

$$\vec{P} - \vec{R}_2 + \vec{R}_3 = 0$$

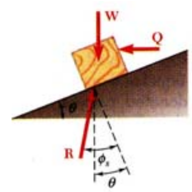
Square-Threaded Screws



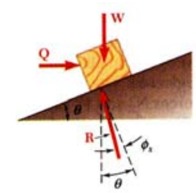
- Square-threaded screws frequently used in jacks, presses, etc. Analysis similar to block on inclined plane. Recall friction force does not depend on area of contact.
- Thread of base has been “unwrapped” and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.
- Moment of force Q is equal to moment of force P. $Q = Pa/r$



- Impending motion upwards. Solve for Q.

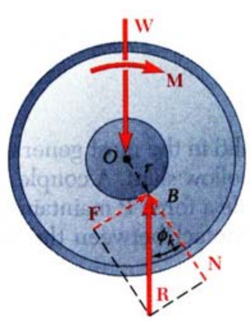


- $\phi_s > \theta$, Self-locking, solve for Q to lower load.



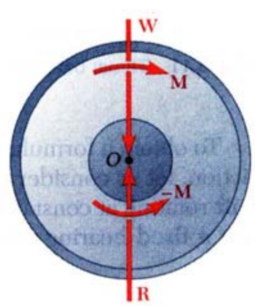
- $\phi_s > \theta$, Non-locking, solve for Q to hold load.

Journal Bearings. Axle Friction

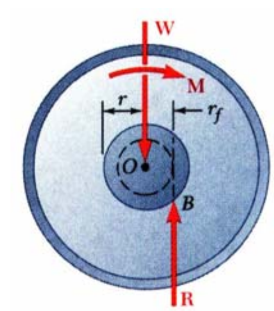


- Angle between R and normal to bearing surface is the angle of kinetic friction φ_k.

$$M = Rr\sin\phi_k$$
$$\approx Rr\mu_k$$



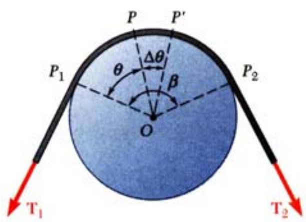
- May treat bearing reaction as force-couple system.

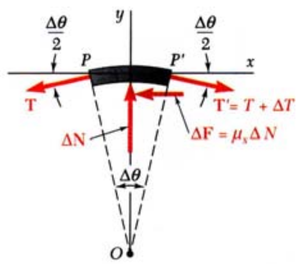


- For graphical solution, R must be tangent to *circle of friction*.

$$r_f = r\sin\phi_k$$
$$\approx r\mu_k$$

Belt Friction





- Relate T_1 and T_2 when belt is about to slide to right.
- Draw free-body diagram for element of belt

$$\sum F_x = 0: \quad (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0: \quad \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

- Combine to eliminate ΔN, divide through by $\Delta\theta$,

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}$$

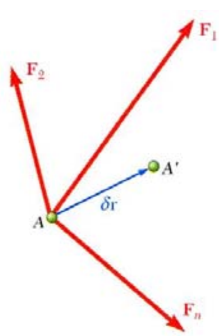
- In the limit as $\Delta\theta$ goes to zero,

$$\frac{dT}{d\theta} - \mu_s T = 0$$

- Separate variables and integrate from $\theta = 0$ to $\theta = \beta$

$$\ln\frac{T_2}{T_1} = \mu_s\beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s\beta}$$

Principle of Virtual Work



- *Imagine* a small *virtual displacement* of a particle which is acted upon by several forces.
- The corresponding *virtual work*,

$$\delta U = \vec{F}_1 \cdot \delta\vec{r} + \vec{F}_2 \cdot \delta\vec{r} + \vec{F}_3 \cdot \delta\vec{r} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right)\cdot\delta\vec{r}$$
$$= \vec{R}\cdot\delta\vec{r}$$

Principle of Virtual Work:

- A particle is in equilibrium if and only if the total virtual work of forces acting on the particle is zero for any virtual displacement.
- A rigid body is in equilibrium if and only if the total virtual work of external forces acting on the body is zero for any virtual displacement of the body.
- If a system of connected rigid bodies remains connected during the virtual displacement, only the work of the external forces need be considered.

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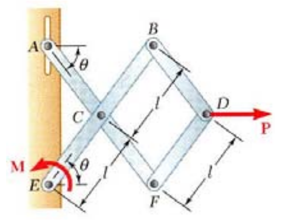
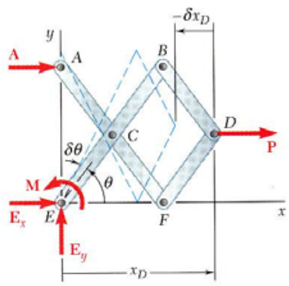
Sample Problem 10.1





Determine the magnitude of the couple M required to maintain the equilibrium of the mechanism.

SOLUTION:

- Apply the principle of virtual work

$$\delta U = 0 = \delta U_M + \delta U_P$$

$$0 = M\delta\theta + P\delta x_D$$

$$x_D = 3l\cos\theta$$

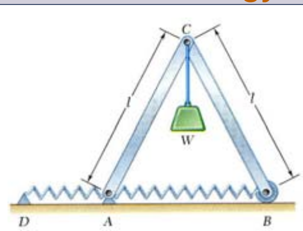
$$\delta x_D = -3l\sin\theta\delta\theta$$

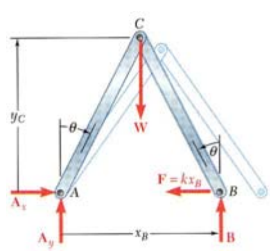
$$0 = M\delta\theta + P(-3l\sin\theta\delta\theta)$$

$$\boxed{M = 3Pl\sin\theta}$$

Vector Mechanics for Engineers: Statics

Potential Energy and Equilibrium (not covered in the final)





- When the potential energy of a system is known, the principle of virtual work becomes

$$\delta U = 0 = -\delta V = -\frac{dV}{d\theta}\delta\theta$$

$$0 = \frac{dV}{d\theta}$$

- For the structure shown,

$$V = V_e + V_g = \tfrac{1}{2}kx_B^2 + Wy_C$$

$$= \tfrac{1}{2}k(2l\sin\theta)^2 + W(l\cos\theta)$$

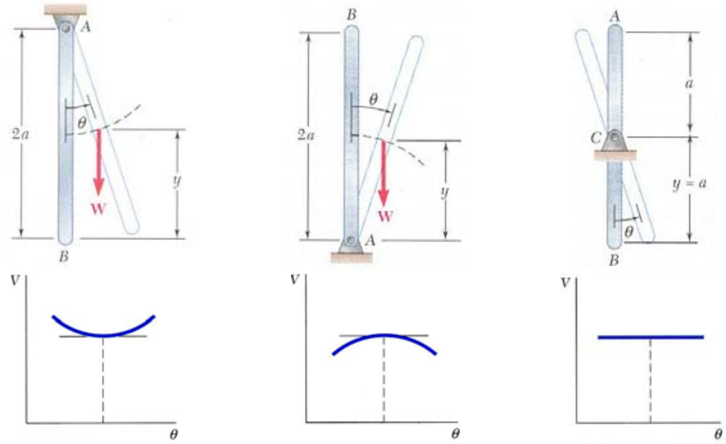
- At the position of equilibrium,

$$\frac{dV}{d\theta} = 0 = l\sin\theta(4kl\cos\theta - W)$$

indicating two positions of equilibrium.

Stability of Equilibrium (not covered in the final)

$$\frac{dV}{d\theta} = 0$$



(a) Stable equilibrium (b) Unstable equilibrium (c) Neutral equilibrium

$$\frac{d^2V}{d\theta^2} > 0$$

$$\frac{d^2V}{d\theta^2} < 0$$

Must examine higher order derivatives.