Phases: from He3 to Nanomechanical Oscillators

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Osheroff65: October 2010

The start of the story

Evidence for a New Phase of Solid He^3 †

D. D. Osheroff, R. C. Richardson, and D. M. Lee
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850
(Received 10 February 1972)

Measurements of the melting pressure of a sample of He^3 containing less than 40-ppm He^4 impurities, self-cooled to below 2 mK in a Pomeranchuk compression cell, indicate the existence of a new phase in solid He^3 below 2.7 mK of a fundamentally different nature than the anticipated antiferromagnetically ordered state. At lower temperatures, evidence of possibly a further transition is observed. We discuss these pressure measurements and supporting temperature measurements.

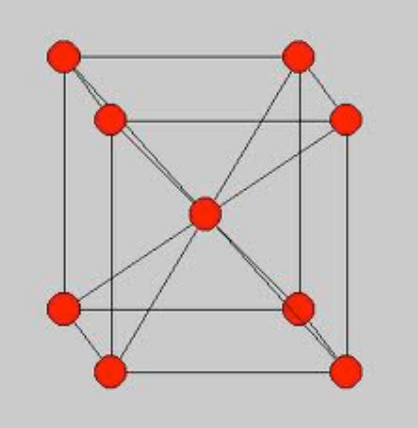
Making amends

Nuclear Antiferromagnetic Resonance in Solid ^{3}He

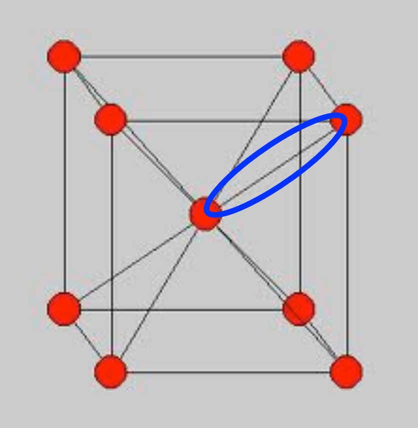
D. D. Osheroff, M. C. Cross, and D. S. Fisher
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 1 February 1980)

Detailed measurements of the low-field antiferromagnetic resonance spectrum of spin-ordered bcc ^{3}He exhibit large shifts from the Larmor frequency, with a zero-field resonant frequency near zero temperature of $\Omega_0/2\pi \simeq 825$ kHz. Analysis of the spectrum leads to stringent constraints on possible sublattice structures. The temperature dependence of Ω_0 shows low-temperature behavior expected from spin-wave theory, and indicates a first-order transition at 1.03 mK.

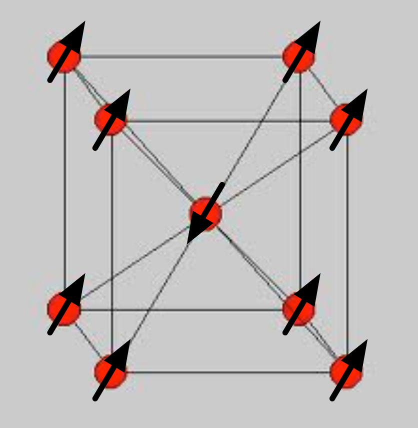
Antiferromagnetic He3: expectations



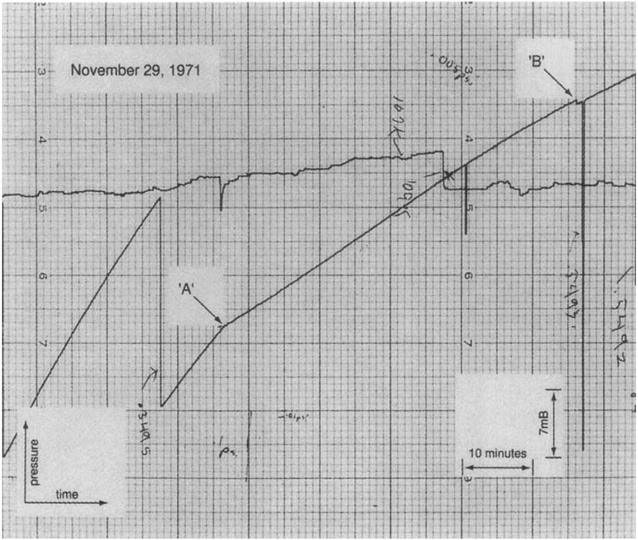
Antiferromagnetic He3: expectations



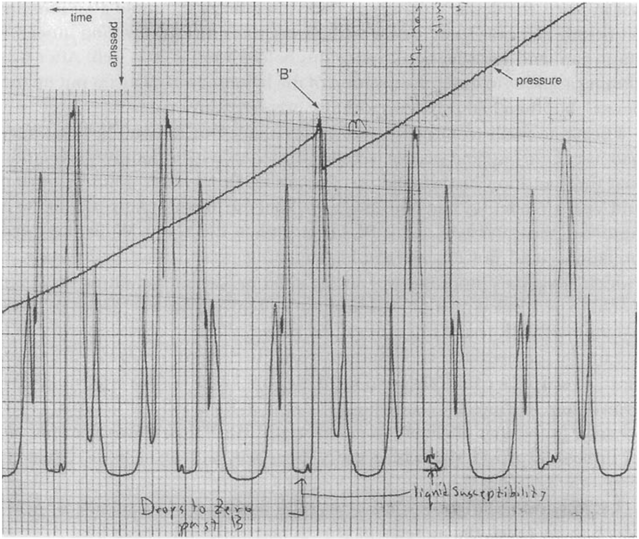
Antiferromagnetic He3: expectations



Doug's data

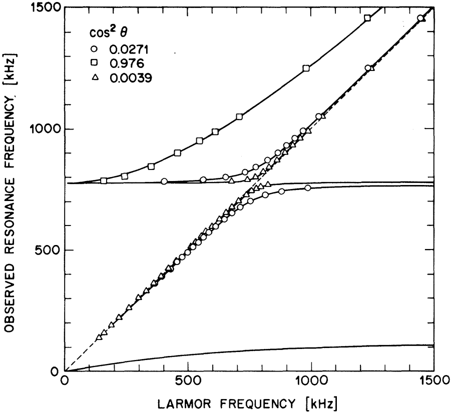


Doug’s data


time
pressure
'B'
pressure
liquid Susceptibility
Drops to zero past B

NMR in solid He3: spectrum

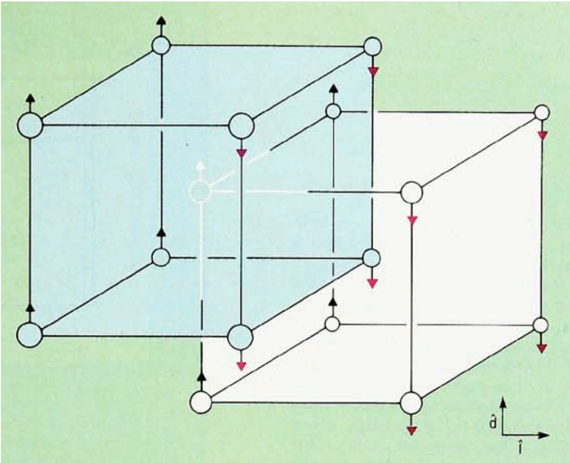
Osheroff, MCC, Fisher (1980)



$$\dot{\vec{S}} = \gamma \vec{S} \times \vec{H} - \lambda(\hat{d} \cdot \hat{l})(\hat{d} \times \hat{l})$$

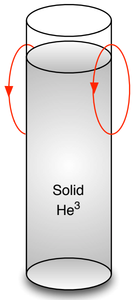
$$\dot{\hat{d}} = \hat{d} \times (\gamma \vec{H} - \gamma^2 \chi_0^{-1} \vec{S})$$

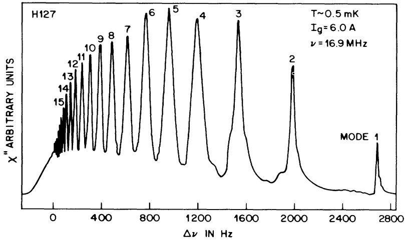
Antiferromagnetic He3: U2D2 Phase

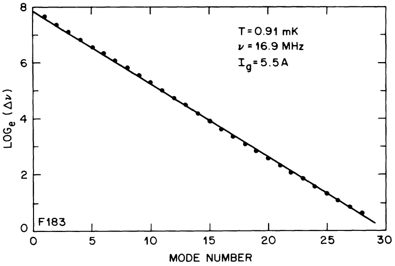


NMR in the high field phase(s)

Osheroff and MCC (1987)



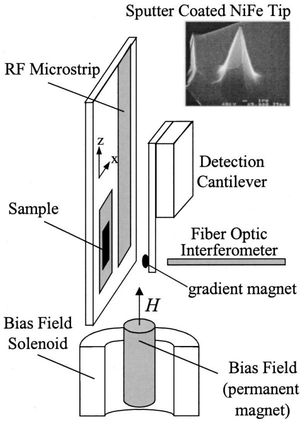


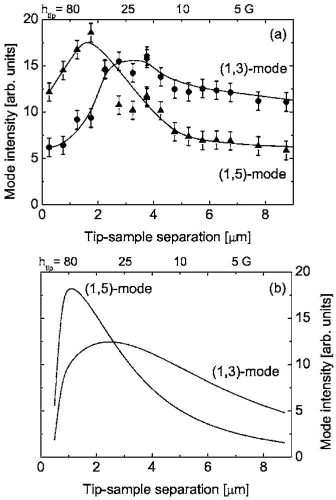


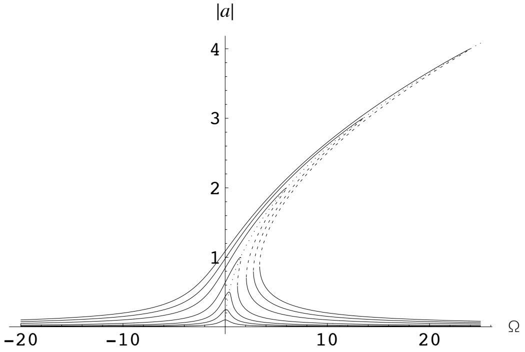
$$-\frac{d^2\psi}{dz^2}+\frac{2\pi M_0}{\Omega-H(z)}\nabla_\perp^2\psi=(-1)\nabla_\perp^2\psi$$

Magnetic Resonance Force Microscopy (MRFM)

Midzor, Wigen, Pelekhov, Chen, Hammel, Roukes, 2000
Urban, Putilin, Wigen, Liou, MCC, Hammel, Roukes, 2007





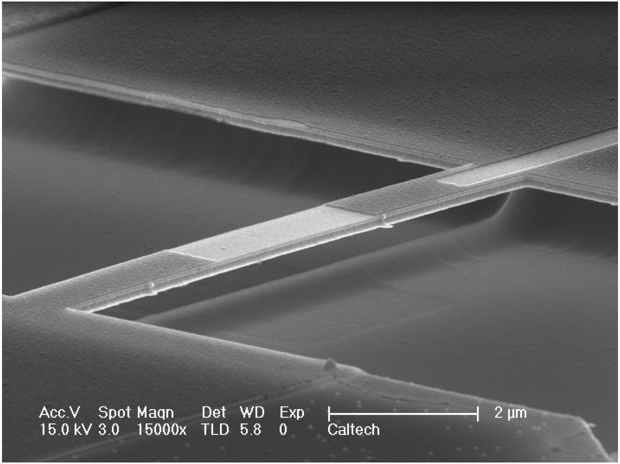


Amplitude v. frequency offset for different drives

Bill Brinkman: Landau and Lifshitz *Mechanics* §29

Nanomechanical systems

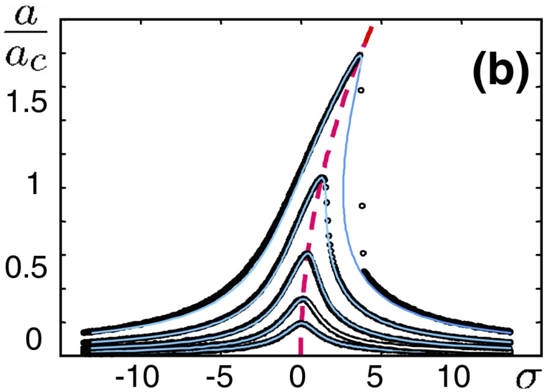
Roukes group, courtesy Rassul Karabalin



Nonlinearity in nanomechanical systems: Duffing response

Kozinsky, Postma, Kogan, Husain, and Roukes, 2007

Doubly-clamped platinum nanowire 2.25μm $\times$ 35nm



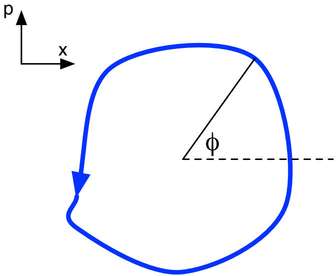
Frequency offset scaled to linear resonance width

Phases in Superfluid He3

- Superfluidity in the A phase
  - Phase is the broken symmetry variable of the quantum mechanical phase
  - $\nabla \times \hat{l}$ gives supercurrents
  - $\nabla\Delta$ gives supercurrents — but reduced by $(T_c/E_F)^2$
  - Exotic Ginzburg-Landau theory
- A and B phases: fun with interfaces (and nucleation)
- NMR as Josephson effect coupling phases of different spin components

Phases in a nanomechanical oscillator

Phase is the broken symmetry variable corresponding to the Hopf bifurcation to oscillations (time translation symmetry)

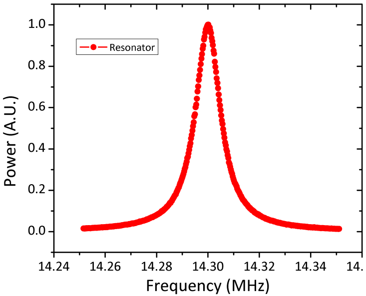


Consequences for frequency stability of oscillators…

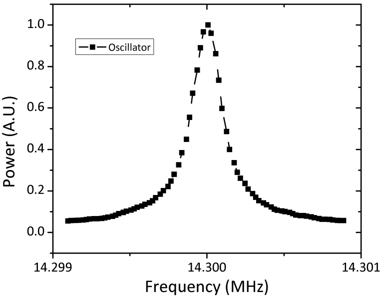
Nanomechanical resonators and oscillators

Roukes group, courtesy Luis Villanueva

Resonator

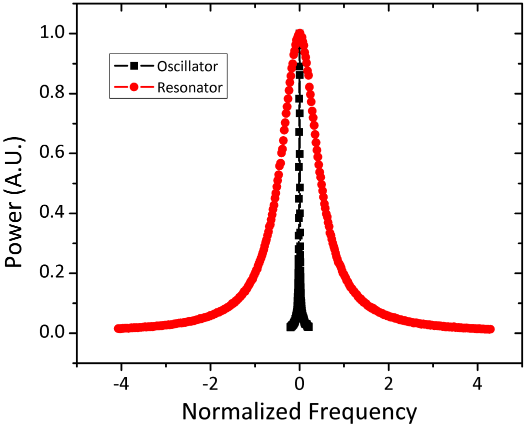


Oscillator



Nanomechanical resonators and oscillators

Roukes group, courtesy Luis Villanueva



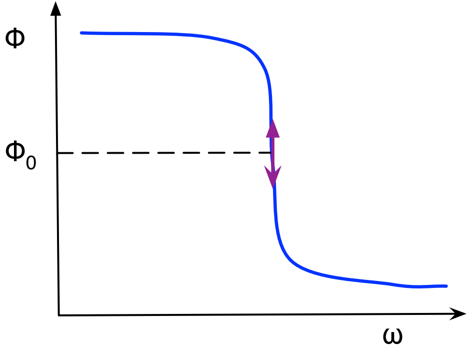
Effective Q enhancement 1200 $\Rightarrow$ 99000

Evading amplifier noise in oscillators

Greywall, Yurke, Busch, Pargellis, and Willett, 1994

For saturated feedback loop: bias is constant *phase*

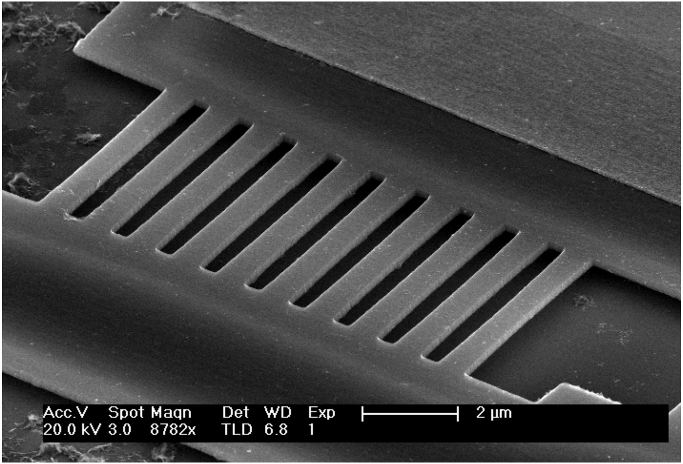
Phase response of driven nonlinear resonator at the Duffing critical amplitude.



Frequency fluctuations of oscillator reduced by tuning to nonlinear critical point

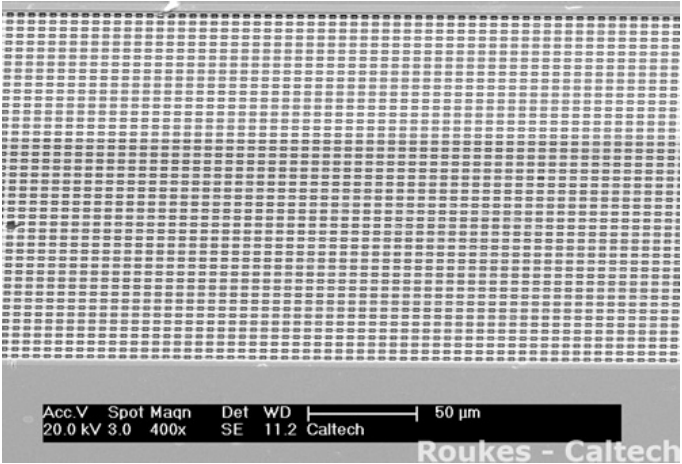
Nanomechanical arrays

Roukes group, courtesy Rassul Karabalin



Nanomechanical arrays

Roukes group



Phases in many nanomechanical oscillators

Oscillator displacement (e.g. amplitude of fundamental mode of beam)

$$u_n = \mathrm{Re}\left[\psi_n(t)e^{i\omega_0 t}\right] \qquad \text{with} \quad \psi_n = |\psi_n|e^{i\phi_n}$$

In long wavelength limit $\psi(\boldsymbol{r})$ satisfies the *Complex Ginzburg-Landau Equation*

$$\frac{\partial\psi}{\partial t} = \psi + (K + i\beta)\nabla^2\psi - (1 + i\omega(\boldsymbol{r}) + i\alpha)|\psi|^2\psi$$

with $\omega(\boldsymbol{r})$ the random component of the frequencies

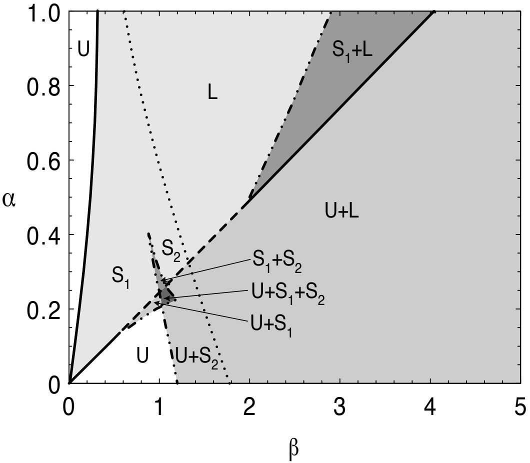
What are the phases (states) of the oscillator phases?

- disorderd
- frequency locked (finite fraction have same frequency)
- phase locked (nonzero $\langle\psi\rangle_{\text{lattice}}$)

Synchronized phases in mean field theory

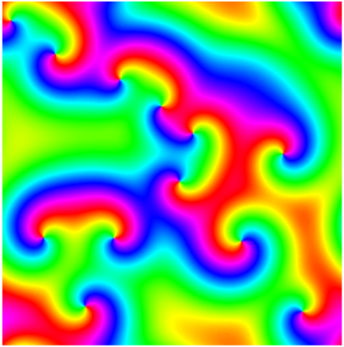
MCC, Rogers, Lifshitz, Zumdieck, 2006

K=0; Top-hat frequency distribution, width = 1

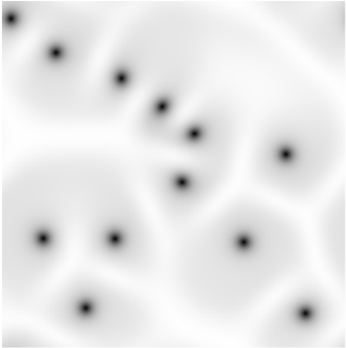


U = unlocked; S_1, S_2 = synchronized; L = locked

Short range model (no randomness)



Phase



Magnitude

$$\frac{\partial \psi}{\partial t} = \psi + (K + i\beta)\nabla^2\psi - (1 + i\alpha)|\psi|^2\psi$$

Happy
65th
Doug!