Physics 161: Homework 9

(March 1, 2000; due March 8)

Problems

1. Find a report in a journal, magazine or book of an experiment describing *control* of a chaotic system. Please do *not* use a reference that has been mentioned in the notes or class, or that you (or others) used in homework 5. Give the reference. Discuss critically what aspects of the understanding of chaotic systems and the analysis of control schemes is used in the method. (If the answer is "none" find a different example!)

2. **OGY Control:** The Hénon map

$$\begin{aligned} x_{n+1} &= y_n + 1 - a x_n^2 \\ y_{n+1} &= b x_n \end{aligned} \quad . \tag{1}$$

with $a = 1.4, b = 0.3$ has a fixed point $\vec{x}_f = (x_f, y_f) \simeq (0.6313, 0.1894)$. The expression for the change in control parameter to be used at the next iteration after $\vec{x}_n = \vec{x}_f + \delta\vec{x}_n$ in the OGY control scheme is Eq.(24.4):

$$\delta p_n = \frac{\lambda_u}{\lambda_u - 1} \frac{\delta\vec{x}_n \cdot \vec{f}_u}{\vec{g} \cdot \vec{f}_u} \tag{2}$$

(see text for notation). Calculate the eigenvalues and eigenvectors of the linearized map at the fixed point, $\vec{g} = d\vec{x}_f/da$, and hence calculate the quantity $(\lambda_u/(\lambda_u - 1))(\vec{g} \cdot \vec{f}_u)^{-1}$ (called *pm* in the output of the *control* demonstration) needed for the control scheme. Compare with the results of the *control* demonstration. Study control to the period 2 orbit in the Hénon map at $a = 1.4, b = 0.3$ using the *control* program. List the position $\vec{x}_p$ of the points in the orbit, the stability eigenvalues at them, the "derivative" $\vec{g} = d\vec{x}_p/da$ and the quantity *pm* (all written out in the applet).

3. **Direct Targeting Control:** Consider the *direct targeting* scheme for controlling to the fixed point $(x_f, y_f) \simeq (0.6313, 0.1894)$ of the Hénon map with $a = 1.4, b = 0.3$ using the single control parameter a.

   (a) Construct the matrices A, B, C and K defined in Eqs.(24.11, 24.18, 24.19) for this scheme in terms of (x_f, y_f).

   (b) Verify that $(A - BK)^2$ gives zero when acting on *any* deviation vector $(\delta x, \delta y)$, so that in the ideal case (e.g. no noise) control is achieved in two iterations!

   (c) Calculate the perturbation in a that would be used for the point $\vec{x}_n = (0.6400, 0.1800)$ and compare with the perturbation that would be used in the OGY control scheme, question 2.