Collective Effects

in

Equilibrium and Nonequilibrium Physics

Website: http://cncs.bnu.edu.cn/mccross/Course/

Goals of Course

1. Show the universality of ideas connecting the physics of equilibrium systems and systems far from equilibrium
2. Discuss methods to understand *collective* effects in wide variety of systems
3. Describe interesting physics such as superfluidity, pattern formation, nanomechanical biodetectors, and morphogenesis
4. Review (with perhaps a different perspective) basic ideas from thermodynamics, statistical physics, hydrodynamics…

Outline

1. Introduction: When is physics simple?
2. The simplest magnet: the Ising model
3. Magnets with continuous symmetry (XY model)
4. Superfluids and superconductivity
5. Onsager theory and the fluctuation-dissipation theorem
6. Hydrodynamics
7. Far from equilibrium
8. Pattern formation
9. Spatiotemporal chaos
10. Pattern formation in biology
11. Collective physics of DNA and proteins
12. Universality and the renormalization group

Today's Lecture

1. Discuss question: "When is physics simple (and interesting)?"

   We will arrive at a number of ingredients:

   - Conserved quantities
   - Broken symmetries (ordering)
   - Near transition (phase transition, instability)
   - ~~More exotic (quantum) ideas~~ …see the talk by Leon Balents
2. Importance of conserved quantities: thermodynamics
3. Broken symmetries and phase transitions: magnets

When is physics simple?

Basic laws of physics (may) be simple, but the systems we look at are complex, e.g.:

- Gas in this room
- Metals, semiconductors, superconductors…
- Atmosphere
- Galaxy
- Cosmology

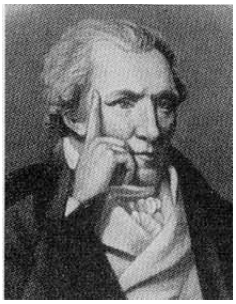
Individual particles do complicated things, but we are mainly interested in aggregate behavior

- Lengths large compared with microscopic ones (e.g., spacing between atoms)
- Time long compared with microscopic ones (e.g., atomic collision time)

When is aggregate behavior interesting?

Equilibrium, Conserved Quantities, and Thermodynamics

Some History

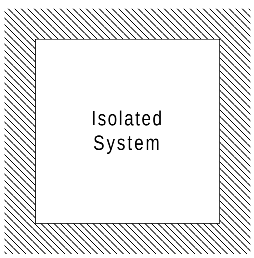


Count Rumford

- 1643-1727 — Isaac Newton: laws of motion
- 1753-1814 — Benjamin Thompson: heat and work
- 1796-1832 — Sadi Carnot: heat engines
- 1822-1888 — Rudolf Clausius: entropy
- 1824-1907 — William Thomson: absolute temperature
- 1839-1903 — Gibbs: thermodynamic potentials, statistical mechanics

Demonstration : few is complicated, many simple

Recipe for Equilibrium



Isolate system of interest from all contact with the remainder of the universe and wait!

Role of Conserved Quantities

How can we prepare two similar systems in the identical equilibrium state? On what properties of the system does the equilibrium state depend?

These must not change even in the very long waiting time to reach equilibrium.

- quantities that are *conserved* by the dynamics of the isolated system
  - ◇ total number of particles
  - ◇ total energy
  - ◇ total momentum or angular momentum in special cases
- may be dependence on the volume of the system
- below certain phase transitions it is appropriate to introduce an *order parameter*

Entropy S and the Condition for Equilibrium

Key idea of the branch of physics we call thermodynamics

- characterizes the equilibrium state and the approach to equilibrium
- useful in understanding the partition of energy into work and heat

Properties

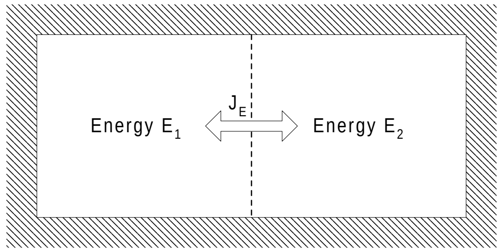
- unique function of the thermodynamic state
- extensive—it increases proportionately as like pieces are added to the system
- additive between systems, so that the entropy of two uncoupled or weakly coupled systems is the sum of the individual entropies

Entropy increases in the approach to equilibrium, and is a maximum in equilibrium (second law of thermodynamics)

For the one-component gas or liquid the entropy is a function of the energy, number of molecules, and volume of the system:

$$S = S(E, N, V)$$

Equilibrium under Energy Transfer



Isolated system divided into two weakly coupled halves or subsystems.

Initially the whole system is in thermodynamic equilibrium.

Take the system away from equilibrium by transferring an energy ΔE from one half to the other.

For weak coupling the time scale for the relaxation will be correspondingly long.

Energy current J_E flows between the subsystems returning the system towards equilibrium, while maintaining the total energy constant, i.e.,

$$E_1 + E_2 = E$$

with the total energy E fixed.

A particular partition of the total energy E_1^{eq}, E_2^{eq} corresponds to the thermodynamic equilibrium state of the whole system:

- entropy S is maximized in the equilibrium state
- total entropy of the system is the sum of the component entropies
- the state of each subsystem is closely approximated as its equilibrium state with energy equal to the instantaneous value of the relaxing energy

  $$S = S_1(E_1) + S_2(E_2) = S_1(E_1) + S_2(E - E_1)$$

  where $S_1(E_1)$ is evaluated as if system 1 were completely isolated with energy content E_1, etc.

Conjugate Thermodynamic Variable

$$S = S_1(E_1) + S_2(E_2) = S_1(E_1) + S_2(E - E_1)$$

Maximization given by differentiating with respect to E_1:

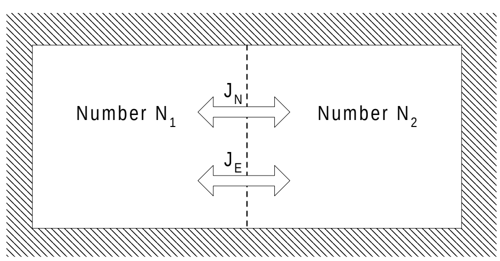
$$T_1 = T_2$$

where a new variable we call the temperature T is defined for each system by

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

Equilibrium of subsystems under the exchange of quantities of the conserved variables leads to the introduction of conjugate thermodynamic variables, in this case the temperature T conjugate to the energy E.

Other Equilibria



Considering the equilibrium under this type of coupling introduces the chemical potential μ conjugate to the number of particles N

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}, \quad \frac{\mu}{T} = -\left(\frac{\partial S}{\partial N}\right)_{E,V}$$

Systems that can exchange particles as well as energy are in equilibrium when both the temperatures and chemical potentials are equal.

Depending on the physical system there might be other conserved quantities and corresponding conjugate fields that are equal in equilibrium, for example a velocity **v** conjugate to the momentum **p** (a fluid will come into equilibrium with containing walls when the velocities are equal), a magnetic field **H** conjugate to a conserved magnetization **M**, etc.

Thermodynamic identity

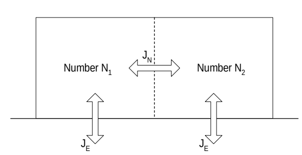
$$dS = \frac{1}{T}dE - \frac{\mu}{T}dN + \frac{P}{T}dV + \sum_i \frac{X_i}{T}dx_i \quad .$$

or

$$dE = TdS + \mu dN - PdV - \sum_i X_i dx_i \quad .$$

with $X_i = (\partial S/\partial x_i)_{E,N,V\ldots}$ or $X_i = (\partial E/\partial x_i)_{S,N,V\ldots}$

Equilibrium in Contact with "Baths"

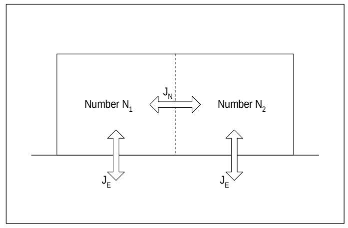


System in contact with a source of energy at constant temperature T, a source of particles at constant chemical potential μ etc., will come to an equilibrium state at long times

- Equilibrium of a system in contact with a bath is given by maximizing (or minimizing) a *new* thermodynamic potential
- For equilibrium in contact with a heat bath the appropriate thermodynamic potential is the free energy $F = E - TS$

$$dF = -SdT + \mu dN - PdV - \sum_i X_i dx_i \qquad \Rightarrow \qquad \mu = (\partial F/\partial N)_{T,V,\ldots}$$

Equilibrium in Contact with "Baths"



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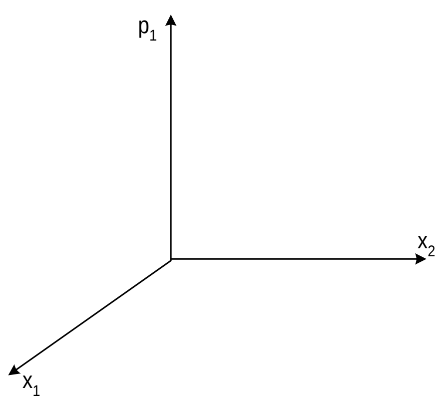
Note that a system in contact with *two* heat baths at the *same* temperature T will again come to the same equilibrium.

However if the two heat baths are maintained at *different* temperatures, there is no possibility of thermal equilibrium. There will be an energy current from one heat bath into the system, which will be transferred to the second bath.

This is a nonequilibrium state.

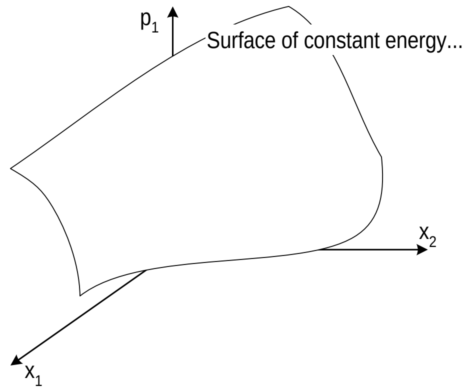
Statistical Mechanics

Statistical Mechanics Interpretation



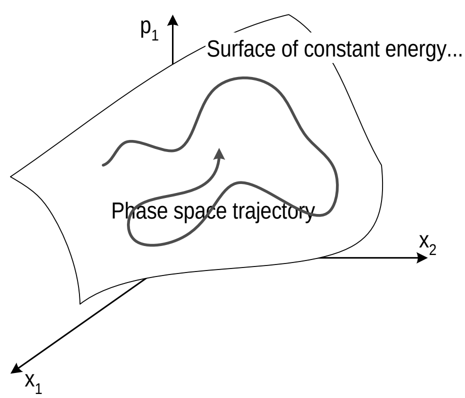
Phase space: $(r_N(t),\, p_N(t)) = (x_1, y_1, z_1, \ldots x_N, y_N, z_N;\, p_{x1}, p_{y1}, p_{z1}, \ldots p_{xN}, p_{yN}, p_{zN})$

Statistical Mechanics Interpretation



Surface of constant total energy, number, etc.

Statistical Mechanics Interpretation



Phase space trajectory $(\mathbf{r}_N(t), \mathbf{p}_N(t))$

Statistical Mechanics Interpretation

Ingredients

- Hamiltonian dynamics: volumes in phase space are preserved by dynamics
- Liouville's theorem: phase space density is constant along a trajectory
- Ergodicity: trajectory visits neighborhood of every accessible point (i.e., on the surface of constant conserved quantities)

Result for equilibrium distribution: probability density is constant over surface of constant conserved quantities

…or take as *fundamental postulate of equal a priori probabilities*

Entropy is given by k_B times the log of the "number of accessible states"

Symmetry and Phase Transitions

Introduction: Magnetism and the Compass

Magnets in History

Compasses were initially used in feng shui in ancient China. The first known use of Earth's magnetic field in this way occurred in ancient China as a spectacle. Arrows were cast similarly to dice. These magnetised arrows aligned themselves pointing north, impressing the audience. The earliest record of use of magnetic lodestone as a direction point was in a 4th century Chinese book: Book of the Devil Valley Master.

The earliest record about the use of compass in navigation was Zhu Yu's book Pingzhou Ke Tan (Pingzhou Table Talks) of AD 1117.

> *The navigator knows the geography, he watches the stars at night, watches the sun at day; when it is dark and cloudy, he watches the compass*

A pilot's compass handbook titled Shun Feng Xiang Song (Fair Winds for Escort) contains great details about the use of compass in navigation.

(from Wikipedia)

Why do compasses work?

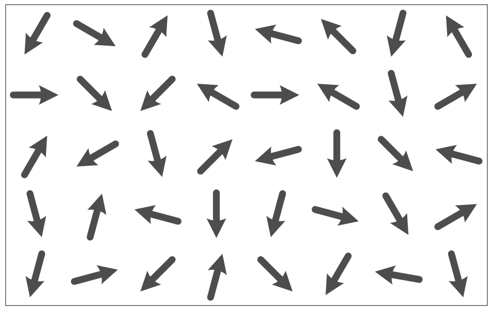
- Earth's horizontal magnetic field (California, USA): $B_E = 24 \times 10^{-6}$ Tesla
- Spin of an electron (Bohr magneton): $\mu_B = 9.3 \times 10^{-24}$ A m^2
- Boltzmann constant: $k_B = 1.4 \times 10^{-23}$ Joules K^{-1}

Compare

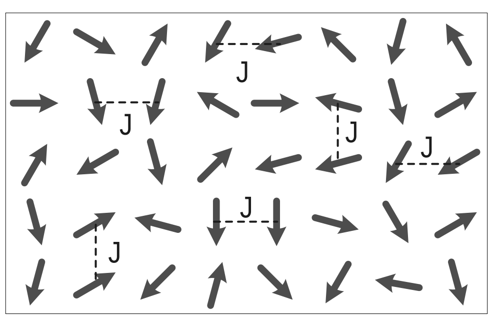
- Magnetic energy aligning spin: $\mu_B B_E = 2.232 \times 10^{-28}$ Joules
- Thermal fluctuation energy: $k_B T = 4.2 \times 10^{-21}$ Joules

Collective effect! Interaction between spins aligns them into one giant spin (total magnetization) which is then aligned by the earth's magnetic field (i.e., magnetic energy is multiplied by of order Avogadro's number 6×10^{23})

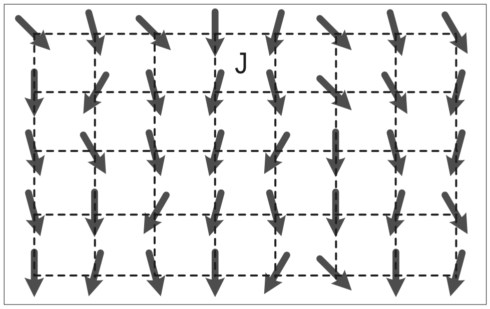
Free Spins



Interacting spins (high T)



Interacting spins (low T)



Ordering and Broken Symmetry

- Neighboring spins tend to align parallel to reduce energy
- Below a transition temperature the (free) energy is minimized if *all* the spins align along a *common* direction
- In a system with rotationally symmetry *any* direction is equally good
- This leads to interesting collective behavior at long length scales
- We will see in the next lecture that there is additional collective behavior at temperatures *near* the transition temperature

The Biggest Question!

Why are the laws of physics simple?

Why are the laws of physics simple?

Some thoughts from the current debate…

- There is one unique “theory of everything” that has simplicity and beauty as an ingredient.
- There is one unique “theory of everything” at very high energies and short length scales, but there are many possible “universes” with different physics of low energy and long length scale behavior. We are necessarily in one for which the long length scale and low energy physics is interesting.
- There are even many “theories of everything” at very high energies and short length scales.
- ……Your guess is as good as mine (or anybody else’s)!

Next Lecture

The simplest magnet: the Ising model

- Broken symmetry and the phase transition
- Mean field theory
- Landau theory