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Two-dimensional electrons in excited Landau levels: evidence for new collective states

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Abstract

Recent magneto-transport experiments on high mobility two-dimensional electron systems have provided strong evidence that a new class of collective electronic states exists at high Landau level occupancy. Among the phenomena observed are strongly anisotropic resistances near half-filling and curious re-entrant integer quantum Hall states in the flanks of high Landau levels. These results have been widely interpreted as evidence for the existence of an unusual class of charge density wave states. This paper emphasizes the underlying arguments for the existence of new phases and reviews the charge density wave interpretation. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Condensed matter physics research, both experimental and theoretical, over the last few years has led to increased appreciation for the complexity of the collective behavior of electrons confined to two dimensions. There is now strong evidence that in the presence of a relatively weak magnetic field perpendicular to the two-dimensional (2D) plane, electron gases in ultra-clean semiconductor heterostructures condense into a class of collective phases unknown just a few years ago. These new phases exhibit electrical transport properties which are qualitatively different from those of the well-studied fractional quantized Hall states at high magnetic fields and the conventional weakly disordered Fermi liquid at zero field. Instead, these properties suggest that the new phases are in fact unusual charge density wave (CDW) states.

The idea that 2D electron systems might exhibit CDW order is not new. Prior to the discovery [1] and explanation [2] of the fractional quantized Hall effect (FQHE) in terms of the Laughlin liquids, Hartree–Fock analyses [3] led to the conclusion that the ground state of the system in the lowest ($N = 0$) Landau level (LL) would be a CDW. On the other hand, this failure of HF theory in the lowest LL does not preclude its success in the higher levels since the small energy differences between various candidate states depend sensitively on the detailed shape of the Landau level wavefunctions. This fact was brought to the fore in 1996 in seminal papers by Koulakov et al. [4–6] and by Moessner and Chalker [7]. In both cases, CDW's were found to flourish in the excited Landau levels even as they fail in the lowest LL. In essence, the key difference between the ground and excited LL's is that the latter possess *nodes* in their wavefunctions while the former does not. These nodes reduce the short-range Coulomb repulsion between electrons and allow the system to satisfy its exchange effect urge to phase separate. As the long-range part of the Coulomb interaction is not affected by the existence of the nodes, a finite length scale is established for the phase separation. This length comes out to be of order the classical cyclotron radius, R_c.

The nature of the CDW phases predicted to exist in high LLs is expected to be quite rich. The early Hartree–Fock work [4–7] predicted that unidirectional 'stripe' phases would be lowest in energy at and near half-filling of the *third* and higher ($N \geq 2$) LL. Away from half-filling a transition to an isotropic 'bubble phase' was suggested. In principle, a hierarchy of bubble phases is expected, terminating in a Wigner crystal deep in the flanks of the LLs. Following the experimental work of Lilly et al. [8] and subsequently Du et al. [9], a storm of new theoretical

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work [10–24] ensued. While this new work has supported much of the basic picture derived from the early Hartree–Fock theories, considerable new insight has been gained. In particular, quantum fluctuations are expected to be quite important and have led to interesting analogies with liquid crystals [10,11].

The purpose of this paper is two-fold. First, we present experimental evidence which shows that something quite unusual occurs when three or more orbital LLs are occupied. We shall argue that this evidence points to the spontaneous development, at very low temperature, of new collective phases of the 2D electron system. We shall also argue that although some (as yet unknown) rotational symmetry-breaking field must exist within high mobility GaAs/AlGaAs heterostructures, it is not responsible for the formation of the new phases but at most serves to *orient* them. Secondly, we shall interpret these new experimental results in terms of the developing theoretical picture of CDW-like states in high LLs. At present, the experimental evidence supporting this interpretation is indirect. Nonetheless, the qualitative agreement between theory and experiment is remarkable and leaves one optimistic that no completely different explanation is lurking in the tall grass. Even so, there are aspects of the data for which no acceptable understanding exists at present. These suggest continued caution with the interpretation as more difficult experiments and demanding comparisons with theory are performed.

While this paper will concentrate on the experimental results of the Caltech/Bell Labs group; very similar findings have been reported by the Columbia/Princeton/Utah/NHMFL/Bell Labs group.

2. The basic observations: anisotropy and re-entrant quantized Hall states

The recent magneto-transport results from high mobility ($\mu \sim 10^7$ cm^2/Vs or higher) 2D electron systems in GaAs/AlGaAs heterostructures fall into two main categories: resistance anisotropies near half filling and re-entrant integer quantum Hall states in the flanks of high Landau levels.

2.1. Anisotropy

Fig. 1 displays the longitudinal resistance of a high mobility 2D electron system in a GaAs/AlGaAs heterostructure at very low temperature ($T = 25$ mK). Two data traces (solid and dashed) are shown; the difference between them lies in the configurations of current flow and voltage measurement used. As the diagrams suggest, the two configurations are merely rotated by 90° relative to one another. At low and high magnetic fields, the traces are quite similar but at certain intermediate fields (e.g. at $B \approx 2.5$ T) they differ enormously. This dramatic effect is an important indicator that something interesting is going on in the 2D system.

As the magnetic field B is increased from zero, both resistances exhibit Shubnikov-de Haas oscillations arising from the discrete Landau level density of states. Above about $B \approx 1$ T, the quantum Hall effect (QHE) regime is entered and the resistances fall to nearly zero over relatively broad regions of field. Although not shown in the figure, the Hall resistance exhibits plateaus in these regions, accurately quantized to h/e^2 divided by an integer. These integer quantized Hall states are centered at magnetic fields corresponding to the complete filling of an integer number of spin-resolved LLs. For the data shown, in which the 2D density $N_s \approx 2.7 \times 10^{11}$ cm^{-2}, the filling factor $\nu \equiv N_s h/eB = 4$ at $B \approx 2.7$ T. At this field both spin sublevels of the ground ($N = 0$) and first excited LL ($N = 1$) are fully occupied; at lower magnetic fields the Fermi level lies in the third ($N = 2$) or higher LL.

The difference between the solid and dashed traces in the figure suggests an anisotropy in the transport coefficients of the 2D system. This anisotropy is largest near half-filling of the Landau levels. Peaks in the resistance R_{xx} are observed for current flowing in one direction, while minima are found in the resistance R_{yy} measured in the orthogonal configuration. At $\nu = 9/2$, corresponding to half filling of the lower spin branch of the $N = 2$ LL, the raw resistance ratio R_{xx}/R_{yy} is about 60 for the sample shown; ratios exceeding 3000 have been recorded [25]. At higher half-odd integers (i.e. $\nu = 11/2$, $13/2$, etc.) a smaller, yet still very significant anisotropy is seen. On the other hand, as the figure clearly shows, no trace of the huge anisotropy at $\nu = 9/2$ is found

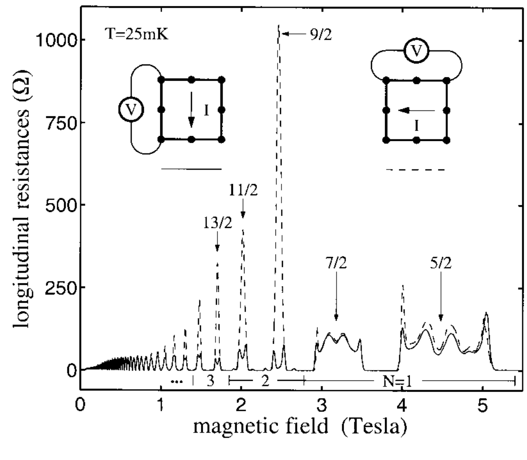

Fig. 1. Longitudinal resistances observed at $T = 25$ mK in a 2D electron gas sample having mobility around 9×10^6 cm^2/Vs. The two traces, dashed and solid, correspond to the resistances R_{xx} and R_{yy} measured with average current flow along the ⟨1-10⟩ and ⟨110⟩ crystal axes, respectively. Contrast the giant anisotropy at $\nu = 9/2$ in the $N = 2$ LL with the essentially isotropic transport near $\nu = 7/2$ in the $N = 1$ LL. The insets depict the arrangement of current flow and voltage measurement. As explained in the text, the solid trace has been multiplied by 0.62. After Lilly et al. [8].

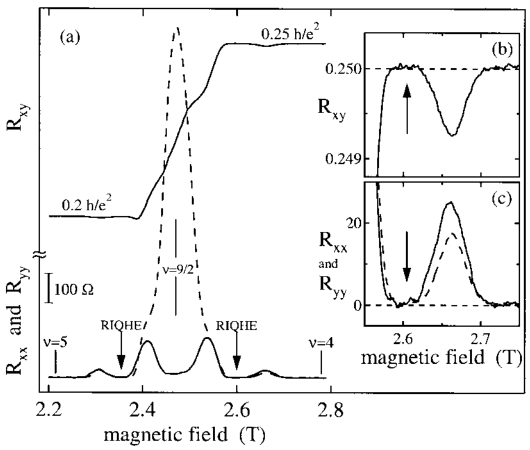

Fig. 2. (a) Longitudinal resistances (dashed curve, R_{xx}; solid curve, R_{yy}) and Hall resistance R_{xy} (upper solid curve) in the $N = 2$ LL at $T = 50$ mK. Arrows indicate location of RIQHE states. Insets (b) and (c) magnify the RIQHE near $\nu = 4 + \frac{1}{4}$. After Cooper et al. [26].

around $\nu = 7/2$. While only 0.7T higher in magnetic field, this filling fraction resides within the $N = 1$ LL.

The anisotropies at $\nu = 9/2$, 11/2, etc. are also strongly temperature dependent, growing rapidly as the temperature is reduced below about 100 mK but eventually saturating at the lowest temperatures. Interestingly, the magnetic field width of the anisotropic region does not fall as the temperature is reduced, in contrast to the expectation for the localization transition between integer QHE states. Finally, the anisotropy is systematically oriented[1] so that the 'hard' resistance R_{xx} always corresponds to average current flow along the $\langle 1{-}10 \rangle$ crystallographic direction, while the 'easy' resistance R_{yy} arises for current flow along $\langle 110 \rangle$.

2.2. Re-entrant quantum hall states

Fig. 2 shows an expanded view of the resistances of a another high mobility 2D electron system over the range $4 < \nu < 5$ in the $N = 2$ LL [26]. Near $\nu = 9/2$ a large anisotropy is clearly evident. Away from half-filling, however, the longitudinal resistances R_{xx} and R_{yy} become roughly equal. Furthermore, both resistances drop to essentially zero in narrow field ranges (indicated by arrows in the figure) located *roughly* at $\nu = 4 + \frac{1}{4}$ and $4 + \frac{3}{4}$. Moving still further into the flanks of the LL the resistances briefly become non-zero again before vanishing into the broad integer quantum Hall states at $\nu = 4$ and $\nu = 5$. The vanishing of the longitudinal resistances at filling factors cleanly separated from the conventional integer QHE suggests the existence of new *fractional* quantized Hall states within the $N = 2$ LL. Examination of the Hall resistance R_{xy} of the sample (also shown in Fig. 2) reveals a surprising result: Well-defined plateaus do appear near $\nu = 4 + \frac{1}{4}$ and $4 + \frac{3}{4}$ where the longitudinal resistances vanish, but they are quantized at the *integer* values of $h/4e^2$ and $h/5e^2$. These anomalous quantum Hall plateaus are separated from the main integer QHE plateaus by narrow ranges in which Hall quantization is lost.

This phenomenon, dubbed the 're-entrant integer quantum Hall effect' or RIQHE, occurs under the same general conditions as does the anisotropy near half-filling. RIQHE states have been clearly seen near $\nu = 4 + \frac{1}{4}$, $4 + \frac{3}{4}$, $5 + \frac{1}{4}$, and $5 + \frac{3}{4}$, in the $N = 2$ LL and analogous locations within the $N = 3$ level. Like the anisotropy, the RIQHE is a very low temperature phenomenon observed only in very high mobility 2D systems. It has not been observed in the $N = 1$ or $N = 0$ LL.

As with conventional QHE states, the longitudinal resistances in the RIQHE are essentially thermally activated. Activation energies or order 1 K have been observed. Interestingly, the peak in the resistance, which separates the RIQHE from the adjacent integer QHE, also shrinks, in height and width, as the temperature falls. While remaining visible down to below 20 mK in the very cleanest samples, in *relatively* dirtier samples the peaks disappear at the lowest temperatures.

The conventional integer quantized Hall effect results from the localization of quasiparticles in the disorder potential in the 2D system. Moving away from precise integer filling increases the density of these quasiparticles and eventually they delocalize and the longitudinal resistance becomes non-zero. The occurrence of the RIQHE, on the other hand, shows that these quasiparticles can localize once again, even as their density is *increasing*. This strongly suggests that an alternative to single-particle localization is operative within the RIQHE. It seems likely that new collective insulating states are present.

3. Intrinsic or extrinsic?

The solid trace in Fig. 1 has been multiplied by a factor of 0.62. This scale factor was applied to match the amplitudes of the very low field ($B < 0.7$ T) Shubnikov–de Haas oscillations of the two traces. This same scale factor leads to good matching of the resistances above about 2.9 T where the Fermi level lies in the $N = 1$ and the ground $N = 0$ LL. Good matching is also obtained in the intermediate field range, but only at temperatures high enough ($T > 150$ mK) to suppress the giant anisotropies at $\nu = 9/2$, 11/2, etc. displayed in Fig. 1. Weak broad-brush 'anisotropies' of this kind are frequently encountered in magneto-transport studies of quantum Hall samples and yet are not well understood. They generally exhibit relatively weak temperature and magnetic field dependences and are not consistently correlated with either the crystalline axes of the GaAs host lattice or with observable features on the sample surface. They may reflect inhomogeneities of the

[1] Provided that no in-plane magnetic field is present.

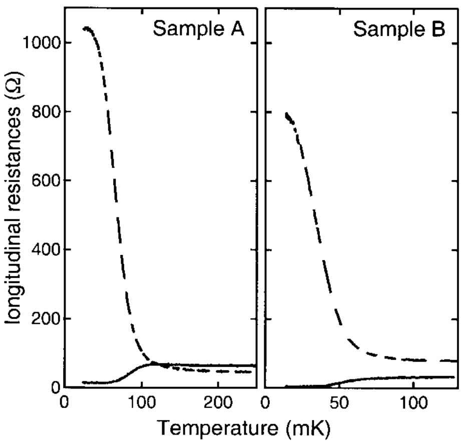

Fig. 3. Temperature dependences of R_{xx} (dashed curves, current flow along ⟨1-10⟩) and R_{yy} (solid curves, current flow along ⟨110⟩) at $\nu = 9/2$ in two samples of different density. Note that while the high temperature 'anisotropy' has an opposite sense in the two samples, the large low temperature anisotropies have the same sense. Densities: Sample A, 2.67×10^{11} cm^{-2}; Sample B, 1.5×10^{11} cm^{-2}.

2D electron gas or be due simply to imprecision in the placement of the ohmic contacts on the sample.

The much larger anisotropies which develop at low temperatures near $\nu = 9/2$, 11/2, etc. are readily distinguished from these small effects. The new anisotropies are extremely sensitive to temperature, magnetic field and filling factor, even within the same Landau level. They are highly reproducible, consistently oriented relative to the crystal axes, and are accompanied by additional intriguing phenomena (e.g. the RIQHE). Nonetheless, these comparisons serve to highlight a fundamental question: Do the anisotropies found in high LLs at very low temperatures represent intrinsic behavior of the 2D electron system or are they the result of some as yet unappreciated extrinsic effect? We believe that the correct answer must be a combination of the two!

There are several important aspects of the high LL transport anisotropies which suggest that they derive from intrinsic collective behavior of 2D electrons:

- *Filling factor dependence.* The anisotropies are strongest at $\nu = 9/2$ and 11/2 in the $N = 2$ LL. They persist, albeit with decreasing strength, into several higher LLs. In contrast, the anisotropy abruptly vanishes if the filling factor is reduced below $\nu = 4$. No significant anisotropies are observed at $\nu = 7/2$ and 5/2 in the $N = 1$ LL, nor at $\nu = 3/2$ and 1/2 in the $N = 0$ ground level. That this sudden transition is associated with filling factor ν rather than magnetic field B has been verified via studies of samples having different densities. (The boundary between anisotropic and isotropic behavior does move, in a fascinating way, if a substantial *in-plane* magnetic field is added to the perpendicular field.)
- *Temperature dependence.* Fig. 3 shows the temperature dependence of R_{xx} and R_{yy} at $\nu = 9/2$ in two samples which have different densities. At high temperatures both samples show a modest residual anisotropy which is only weakly temperature dependent. On the other hand, at low temperatures both rapidly develop a very large anisotropy. Note, however, that while the high temperature anisotropy has a different sense relative to the crystal axes in the two samples, the giant low temperature effects are oriented in the same way. As already mentioned, at low temperatures the hard resistance direction is always[1] along ⟨1-10⟩.
- *In-plane magnetic field dependence.* Quite surprisingly, small in-plane magnetic fields can profoundly alter the low temperature anisotropies at $\nu = 9/2$, 11/2, etc. [27,28]. If properly oriented relative to the crystal axes, even a small in-plane field ($B_{\parallel} \sim 0.5$T) is sufficient to *interchange* the hard and easy directions of the anisotropy. This behavior is illustrated in Fig. 4 for $\nu = 9/2$.

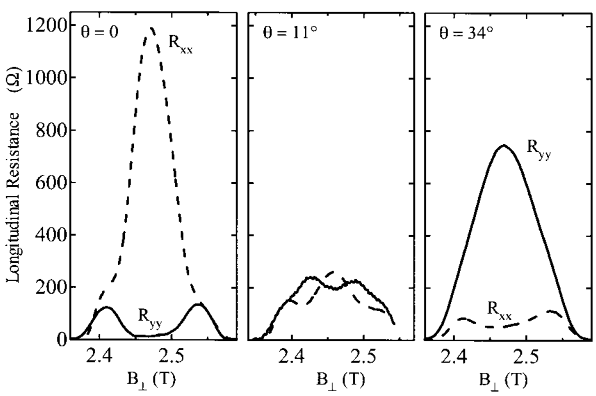

Fig. 4. Interchange of resistance anisotropy at $\nu = 9/2$ at $T = 50$ mK due to an in-plane field. The in-plane field is produced by tilting the sample. At $\theta = 0$, $B_{\parallel} = 0$; at $\theta = 11°$, $B_{\parallel} = 0.48$ T; at $\theta = 34°$, $B_{\parallel} = 1.66$ T. At $\nu = 9/2$ the perpendicular field is fixed at $B_{\perp} = 2.47$ T. After Lilly et al. [27].

Similar behavior is seen at $\nu = 11/2$, 13/2, 15/2, etc. In all cases, the in-plane field eventually leads to the hard direction being parallel to the field.
- *Association with other novel phenomena.* Although the relation between the anisotropies near half filling of high LLs and the RIQHE states in the flanks of the same levels is not empirically obvious, the fact that they occur under very similar conditions is not likely an accident. The RIQHE states serve as strong and independent indicators that novel physics sets in at the third and higher LLs.

It is easy to imagine that some extrinsic effect might be responsible for small 'anisotropies' which exhibit relatively little magnetic field, filling factor, or temperature dependence. Weak wafer-scale density gradients, misaligned contacts, unintentional ordering of the dopant ions, anisotropic roughness at the sample's surface or buried interfaces, and terrace formation due to miscut of the nominally ⟨001⟩-oriented GaAs substrate wafer from its parent boule, are among the many potential sources of such small anisotropies. It is much harder to imagine, however, that effects such as these could produce the extreme sensitivity to filling factor, temperature and in-plane magnetic field that the giant anisotropies at $\nu = 9/2$, 11/2, etc. exhibit. The stark difference between $\nu = 9/2$ and $\nu = 7/2$, for example, cannot be readily explained by simply asserting that there is some preferential orientation within the quasi-random disorder potential seen by the electrons. Instead, this strong filling factor dependence clearly suggests that the nature of the electron gas near half filling of the third and higher Landau level is different than in the lower levels. The rapid development at very low temperature of the anisotropies (and the RIQHE) also suggests an intrinsic effect. While perfectly consistent with a subtle electron–electron interaction phenomenon (e.g. the fractional QHE), the chance that this sharp temperature dependence instead reflects some spontaneous ordering of the quasi-random potential seems rather remote.

Nonetheless, there is clearly a role for extrinsic[2] effects. The observed systematic anisotropy of the low temperature resistance of 2D electron systems at high LL occupancy *necessarily* implies the existence of some symmetry breaking field in the sample. In our view, the role of this field is not to generate the anisotropic state itself, but to *orient* it. Ferromagnetism provides a good analogy: the magnetism itself is an interaction effect but the orientation of the moment is determined by extrinsic effects.

The fact that the high LL anisotropies are observed in macroscopic samples (typically $5 \times 5\ \mathrm{mm}^2$) and are always oriented in the same way relative to the crystal axes, suggests a simple overarching effect and various candidates have been suggested. For example, the lack of inversion symmetry at typical heterointerfaces between GaAs and $Al_xGa_{1-x}As$ opens up the possibility for the loss of 4-fold symmetry of the in-plane band dispersion [29]. Recent experiments, however, have shown that the same anisotropies are observed in samples in which the electrons are confined in a square quantum well which was designed to be as symmetric as possible [30]. Another possibility involves the anisotropic kinetics of crystal growth by molecular beam epitaxy. These effects can lead to anisotropic roughness on the sample surface and presumably at its buried interfaces. Indeed, atomic force microscopy studies have revealed preferential orientations of micron-sized surface features on most GaAs/AlGaAs samples. While some [31] have reported that these features are consistently oriented relative to the GaAs crystal axes, we find that this is not the case [30]. Samples which clearly exhibit high LL resistance anisotropies oriented in the same way relative to the underlying crystal axes can nonetheless display obviously "orthogonal" surface roughness features.

At present, the mechanism which orients the transport anisotropy remains a mystery. In spite of this, there is interesting evidence that the effect is very weak. Jungwirth et al. [15] have performed detailed calculations of the effect of an in-plane magnetic field $B_\parallel$ on putative unidirectional charge density wave states at $\nu = 9/2$, 11/2, etc. Deferring for the moment the question of whether this is a good starting point, these calculations reveal that the in-plane field does exert an orienting force on the stripes which tends to align them perpendicular to field. Since it is reasonable, within the CDW interpretation, to assume the hard transport direction is across the stripes, Jungwirth's result is in agreement[3] with the experimental finding that the hard direction at large $B_\parallel$ is parallel to the in-plane field. Combining the experimental observation (see Fig. 4) that the anisotropy at $\nu = 9/2$ switches direction when $B_\parallel \approx 0.5$ T is applied along the ⟨110⟩ direction with the quantitative calculation of the field-induced anisotropy energy, we estimate a native anisotropy field of only about 1 mK per electron. Since this is much smaller than the temperature (∼100 mK) at which the anisotropy develops, it supports our conclusion that the transport results at $\nu = 9/2$, 11/2, etc. stem from new collective broken symmetry states which are oriented by a weak native anisotropy field within the host material.

4. Stripes and bubbles

The CDW picture originally advanced by Koulakov et al. [4–6] and by Moessner and Chalker [7] provides a framework for understanding many of the experimental results described above. In a window around half filling these Hartree–Fock theories predict that the ground state of a

[2] By extrinsic, we mean any effect not present in an ideal 2D electron system which has complete rotational and translational symmetry in the plane.

[3] Jungwirth's calculations have also had notable success in explaining the complex behavior of anisotropic states in quasi-2D systems which have two subbands occupied. See Ref. [32].

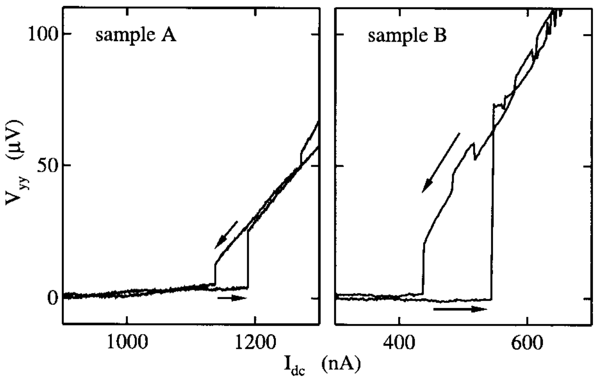

Fig. 5. Discontinuous, and hysteretic, dc current–voltage characteristics observed at $T = 25$ mK in the center of the RIQHE near $\nu = 4 + \frac{1}{4}$. After Cooper et al. [26].

2D electron system in the third and higher LLs will be a unidirectional CDW. The period of the CDW is of order the classical cyclotron radius: $\lambda \approx 3R_c$. For the data in Fig. 1, $\lambda \approx 1000$ Å at $\nu = 9/2$. In terms of filling factor these CDWs are sharply defined: at $\nu = 9/2$, for example, the system forms alternating stripes of $\nu = 4$ and $\nu = 5$. Since the CDW period is comparable to R_c, the actual charge density modulation is small, typically only a few percent. On moving away from half filling one set of stripes widens while the other narrows. Eventually a transition to a triangular CDW composed of multi-electron bubbles is anticipated. Far in the flanks of the LL this bubble phase should become equivalent to a Wigner crystal (i.e. a triangular lattice of 1-electron bubbles.)

It is certainly plausible that if a stripe phase exists near half filling and is somehow consistently oriented relative to the GaAs crystal axes, anisotropic transport would result. Presumably the conductivity along the stripes would be high while that perpendicular to them would be low. This translates into high resistance perpendicular to the stripes and low resistance parallel to them.[4] Furthermore, the theoretical prediction that CDWs are the ground state of 2D electron system only in third and higher Landau levels is in agreement with the experimental observation that the anisotropy abruptly disappears on entering the $N = 1$ second LL.

A large theoretical literature has developed in the aftermath of the experimental results of Lilly et al. [8,26,27], Du et al. [9], and Pan et al. [28]. Finite size exact diagonalization calculations by Haldane et al. [14] lent considerable support to the early Hartree–Fock results. Going beyond Hartree–Fock, Fradkin and Kivelson [10,11], adopting the nomenclature of liquid crystals, suggested that the translational symmetry which is broken in one direction in a *smectic* stripe phase may be restored by quantum fluctuations. Rather than producing an isotropic system, a quantum *nematic* phase with broken rotational symmetry results.

Fertig [12,13], and subsequently MacDonald and Fisher [18], have pointed out that uniform parallel stripes are unstable to density modulations along their length. The resulting smectic crystal would, in principle, be insulating. MacDonald and Fisher, however, predict that the melting temperature for this crystal is unattainably low near half filling while Fertig suggests the possibility that the crystal may always be melted by quantum fluctuations. In either case there is little doubt that the finite temperature transport coefficients would be highly anisotropic.

Away from half filling the experiments reveal that the resistance becomes isotropic again and, near quarter filling, re-entrant integer QHE states develop. These facts are consistent with the Hartree–Fock predictions of a transition to isotropic triangular bubble phases in the flanks of the $N \geq 2$ Landau levels. A triangular CDW is much more likely to be pinned, and therefore insulating, than is a unidirectional stripe CDW. The RIQHE might thus be due to the pinning of a triangular CDW consisting, possibly, of 2-electron bubbles. We have pursued this idea by searching for transport non-linearities in the RIQHE states [26]. Fig. 5 shows that as the current through the sample is increased, the RIQHE breaks down in a dramatic way: Discontinuous, hysteretic, and noisy transitions between insulating[5] and conducting states are seen. These transitions have so far been detected near $\nu \approx 4 + 1/4$, $4 + 3/4$, $5 + 1/4$ and $5 + 3/4$, but are not seen in the adjacent conventional integer QHE states. Although these discontinuous onsets of conduction strongly suggest collective depinning, other QHE breakdown mechanisms may also be at work [33]. At the same time, new exact diagonalization results by Rezayi et

[4] The conversion between conductivity and resistivity for an anisotropic system is of the form $\sigma_{xx} = \rho_{yy}/(\rho_{xy}^2 + \rho_{xx}\rho_{yy})$. Thus, large σ_{xx} is associated with large ρ_{yy}.

[5] Only the electrons in the uppermost LL are insulating; current is readily carried by the edge states of the filled Landau levels beneath the Fermi level.

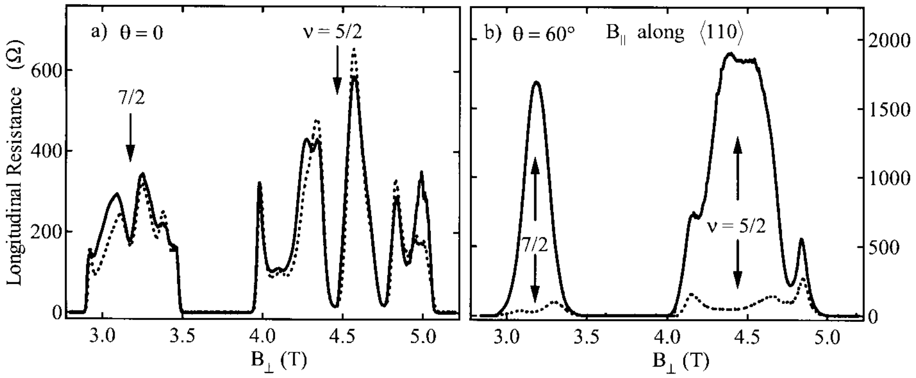

Fig. 6. Transport at $T = 20$ mK in the $N = 1$ Landau level, with and without an in-plane magnetic field. At zero tilt ($B_{\|} = 0$) a strong $\nu = 5/2$ FQHE state is seen and the resistance is essentially isotropic. (Dashed curves, R_{xx}, current flow along ⟨1-10⟩; solid curves R_{yy}, current flow along ⟨110⟩). Tilting the sample by 60° destroys the $\nu = 5/2$ and 7/2 states and renders the resistance highly anisotropic. For these data the in-plane field is directed along ⟨110⟩; at $\nu = 5/2$ $B_{\|} = 7.7$ T. Note the different vertical scales. After Lilly et al. [27].

al. [24] have shown very clear evidence for the formation of a triangular 2-electron bubble phase near $\nu \approx 4 + 1/4$.

Transport theories of the stripe phases have begun to appear recently. MacDonald and Fisher [18] have developed a model in which the edge states within a system of parallel stripes are viewed as weakly coupled Luttinger liquids. Von Oppen et al. [17] have examined the transport characteristics of a highly disordered system of stripes. Both of these approaches have led to rather remarkable universal predictions about the conductivity product[6] $\sigma_{xx}\sigma_{yy}$. These predictions are in surprisingly good, if preliminary, agreement with experiments [34].

5. The $N = 1$ Landau level

The $N = 1$ Landau level has been enticingly difficult to understand for many years [35]. Although fractional QHE states are observed in this first excited LL, there are very few. Most famous is the even-denominator state at $\nu = 5/2$ (and its conjugate at $\nu = 7/2$). These states have resisted explanation since their discovery in the late 1980s. At the moment there is a growing feeling that the exotic Moore-Read Pfaffian paired state [36] may be the winner, but this enthusiasm rests almost exclusively on theoretical arguments.

In the absence of in-plane magnetic fields, the giant low temperature transport anisotropies we have been discussing are not seen in the $N = 1$ LL. However, it has been long known that even small in-plane fields rapidly suppress the $\nu = 5/2$ FQHE [35]. What was not realized until recently [27,28] was that larger in-plane fields produce highly anisotropic transport in the $N = 1$ LL. Fig. 6 shows transport data from the $N = 1$ LL which clearly reveals this transition. As with the $N \geq 2$ LLs, the hard transport direction is parallel to the in-plane field. This anisotropy in the $N = 1$ LL grows steadily as the temperature is reduced, although not as rapidly that shown in Fig. 3 for $\nu = 9/2$ at $B_{\|} = 0$. Conceivably, the 5/2 FQHE state is close in energy to a CDW type of state similar to those in the higher levels and the in-plane field can tip the balance. This scenario has received significant theoretical support from recent exact diagonalization studies [20]. So far there are no signs, from either theory or experiment, that anisotropic phases can infect the $N = 0$ ground LL.

6. Puzzles

As encouraging as the CDW model and its enhancements are, there are aspects of the experiments which are not understood. A few of these are listed here.

- *Native anisotropy field.* The symmetry breaking field which consistently orients the transport anisotropy relative to the crystal axes remains unknown. Until this problem is solved, the degree to which extrinsic effects influence the formation of the anisotropic phases cannot be assessed quantitatively.
- *Spin sublevel dependences.* Although not discussed here, essentially every feature of the high Landau level problem shows an unexplained dependence on spin sublevel. For example, when in-plane fields are applied the behavior of the $\nu = 9/2$ and 11/2 states are reproducibly different. Oddly, the 9/2 state in the $N = 2$ LL more closely resembles the 13/2 state in the $N = 3$ LL. Similarly, 11/2 resembles 15/2.
- *In-plane magnetic fields.* Related to the previous point, there are significant puzzles remaining in the tilted field

[6] The anisotropy *ratio* is not universal!

data. In particular, the complex behavior observed [27,28] at $\nu = 9/2$ and 13/2 is not explained by existing theories [15,16].

- *Non-linear transport at half filling.* Lilly et al. [8] noted that the longitudinal resistances at half filling of the $N \geq 2$ LLs exhibit a smooth non-linearity which is not consistent with heating. Increasing the drive current actually increases the observed anisotropy. While there have been suggestions [12,13,18] for the origin of this effect, it remains very poorly understood.
- *Role of edges.* To our knowledge, no attention has been paid to the role of the edge channels of the submerged Landau levels. While there is no evidence that they are fundamentally important, quantitative analyses of the transport [18,19,33] require they be carefully studied.

At present, these puzzles do not seem likely to upset the basic CDW picture of high Landau level physics. Nonetheless, if their final resolutions cannot be accounted for within the CDW picture, alternative explanations for the transport results discussed here will have to be sought. For example, collective Laughlin-like states having broken rotation symmetry were proposed a few years ago as candidates for fractional quantum Hall states in thick 2D systems [37]. More recently, composite fermion states analogous to those present in the lowest LL were suggested [31] as being relevant in high Landau levels.

7. Conclusions

The recently discovered transport anomalies in high Landau levels have revealed a entirely new class of collective states of 2D electron systems. At the qualitative level, the data are well accounted for by the developing picture of charge density wave and/or liquid crystal states. Nonetheless, there is much more to be done before a deep understanding of the phenomenology of the experiments will exist. Beyond additional theoretical work, it seems obvious that new experiments, distinct from electrical transport, are urgently needed. The situation is reminiscent of the first years after the discovery of the FQHE: perhaps this new field will display a similar longevity.

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