Ph125c Homework #2, due Tuesday April 11 at noon

Problems 1 and 2: **the Stark Effect for the hydrogen atom**

Consider a hydrogen atom placed in a uniform constant electric field E parallel to the z coordinate axis. We wish to compute corrections to the energy spectrum of the non-relativistic Coulomb Hamiltonian

$$H_0 = \frac{p^2}{2m} - \frac{e^2}{r}.$$

For the purposes of this homework you may ignore fine and hyperfine structure, as well as both the electron and nuclear spins.

The perturbation Hamiltonian that describes the effect of the applied electric field is

$$W_S = -q\vec{E} \bullet \vec{r},$$

where q is the charge of the electron, $\vec{E} = E\hat{z}$ is the applied electric field, and $\vec{\mathbf{r}}$ is the (vector) position operator for the electron.

1. (3 points) Show that the leading order correction to the energy of the $1s$ state is negative, and quadratic in E.

2. (6 points) Compute the leading order correction to the energies of states in the $n = 2$ shell of atomic hydrogen. You may leave your answer in terms of the matrix element

$$\gamma E \equiv \langle n = 2, l = 1, m = 0 | W_S | n = 2, l = 0, m = 0 \rangle.$$

3. (6 points) Read the introduction of "The hydrogen atom, a tool for metrology" by B. Cagnac *et al*, Rep. Prog. Phys. **57**, 853-893 (1994) (available online via library.caltech.edu). In your own words, state what the Rydberg constant is and why is it an important quantity to measure experimentally. Somewhere between 0.5 and 1.0 pages would be a good length for your answer.

4. (6 points) The ^{133}Cs atom is hydrogenic (one valence electron in an $n = 6$, $l = 0$ state) but has a total nuclear spin $I = 7/2$. Without computing explicit values for the various shifts and splittings, sketch what you expect to see for the relative energies of all states with $n = 6$ and $l = 0$ in a weak magnetic field. Explain the qualitative features of your diagram.