Maple worksheet by E.S. Phinney 1/28/2004 ACM95b/100b handout.
Solution to the delay differential equation

$$\frac{d\Delta}{dt} = -\lambda\Delta(t-1) \tag{1}$$

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>  finddelta := proc(lambda) :  qeqn := q ->
>  sin(q)/q*exp(q*cos(q)/sin(q)) -1/lambda:  deltaf :=0:  print(‘solution
>  of d Delta(t)/dt = -lambda*Delta(t-1)‘):  print(‘for lambda=‘,
lambda):
>  print(‘Laplace transform’s poles are at‘):  for x in [0,2*Pi,4*Pi]
>  do
>  xl := evalf(x+0.1):
>  xu := evalf(x+2*Pi-0.1):
>  qq := fsolve(qeqn(q),q,xl..xu):
>  pp :=evalf(log(lambda*sin(qq)/qq)):
>  sr :=pp+I*qq:
>  term := evalc(2*Re(lambda*exp(sr*t)/(sr*(1+sr)))):  deltaf
>  :=deltaf+term:
>  print(‘sr=‘, sr, ‘and conjugate‘):
>  od; print(‘giving 3 slowest decaying terms of‘):  print(‘ delta(t)=‘,
>  deltaf); plot(deltaf(t),t=5..25, labels=[‘t‘,‘Delta(t)‘]); end;
```

```
> finddelta(2);
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$$\textit{solution of d Delta(t)/dt = -lambda * Delta(t - 1)}$$

$$\textit{for lambda} =, 2$$

$$\textit{Laplace transform's poles are at}$$

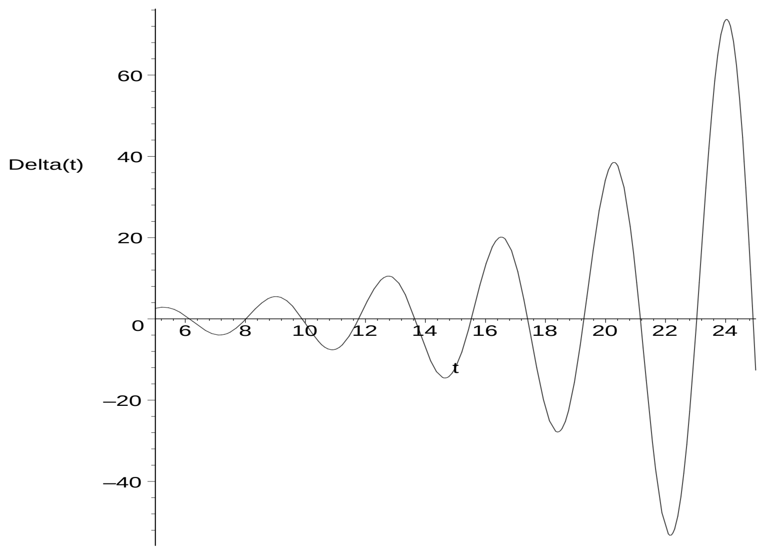
$$sr =, .1728160027 + 1.673686414\,I,\ \textit{and conjugate}$$

$$sr =, -1.360749425 + 7.678589080\,I,\ \textit{and conjugate}$$

$$sr =, -1.955456866 + 13.99837337\,I,\ \textit{and conjugate}$$

$$\textit{giving 3 slowest decaying terms of}$$

$$\begin{aligned}
\textit{delta}(t) =, &-.8790236356\,e^{(.1728160027\,t)}\cos(1.673686414\,t)\\
&+ .7618522186\,e^{(.1728160027\,t)}\sin(1.673686414\,t)\\
&- .06508480242\,e^{(-1.360749425\,t)}\cos(7.678589080\,t)\\
&- .01471417253\,e^{(-1.360749425\,t)}\sin(7.678589080\,t)\\
&- .01973933325\,e^{(-1.955456866\,t)}\cos(13.99837337\,t)\\
&- .004144240478\,e^{(-1.955456866\,t)}\sin(13.99837337\,t)
\end{aligned}$$



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> finddelta(1.571);
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$$\textit{solution of d Delta(t)/dt} \;=\; -\textit{lambda} * \textit{Delta}(t-1)$$

$$\textit{for lambda} =,\ 1.571$$

$$\textit{Laplace transform's poles are at}$$

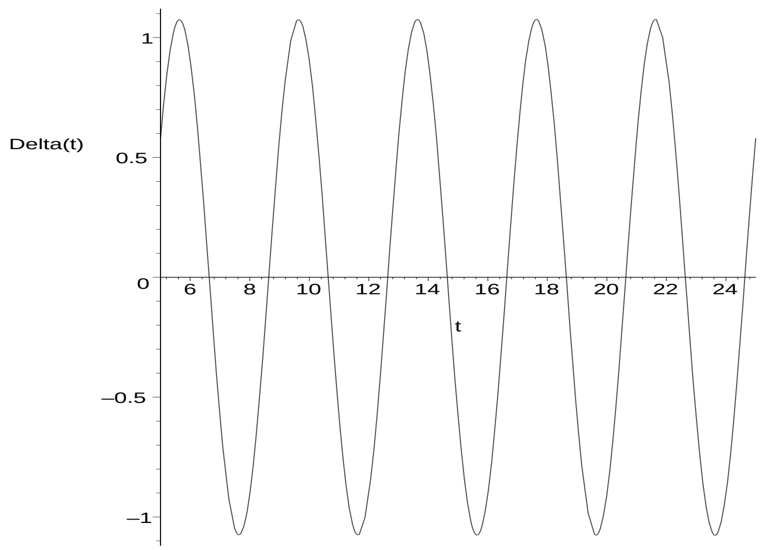
$$sr =,\ .00009226174375 + 1.570855060\,I,\ \textit{and conjugate}$$

$$sr =,\ -1.604159929 + 7.647209125\,I,\ \textit{and conjugate}$$

$$sr =,\ -2.198212187 + 13.98121751\,I,\ \textit{and conjugate}$$

$$\textit{giving 3 slowest decaying terms of}$$

$$\begin{aligned}
&\textit{delta}(t) =, -.9060238606\, e^{(.00009226174375\, t)} \cos(1.570855060\, t)\\
&+ .5768991380\, e^{(.00009226174375\, t)} \sin(1.570855060\, t)\\
&- .05029654746\, e^{(-1.604159929\, t)} \cos(7.647209125\, t)\\
&- .01476913063\, e^{(-1.604159929\, t)} \sin(7.647209125\, t)\\
&- .01536176582\, e^{(-2.198212187\, t)} \cos(13.98121751\, t)\\
&- .003782768736\, e^{(-2.198212187\, t)} \sin(13.98121751\, t)
\end{aligned}$$



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> finddelta(1);
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$$solution\ of\ d\ Delta(t)/dt\ =\ -lambda * Delta(t-1)$$

$$for\ lambda =, 1$$

$$Laplace\ transform's\ poles\ are\ at$$

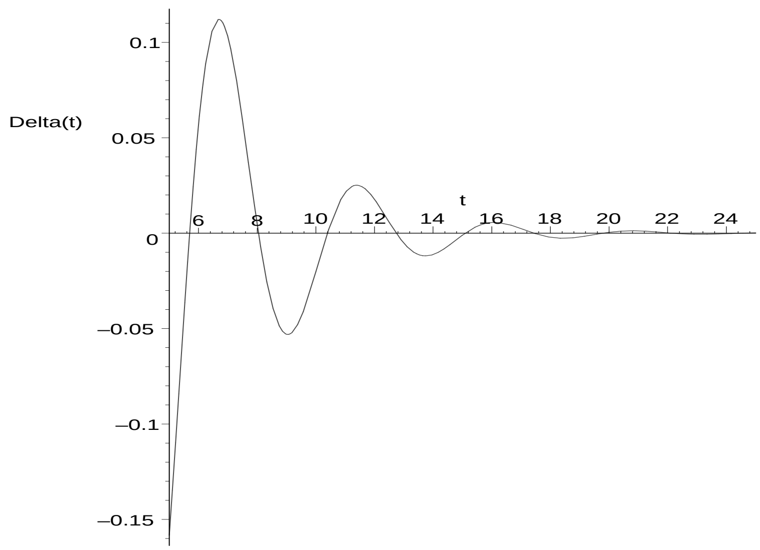
$$sr =, -.3181315050 + 1.337235701\, I,\ and\ conjugate$$

$$sr =, -2.062277729 + 7.588631178\, I,\ and\ conjugate$$

$$sr =, -2.653191974 + 13.94920833\, I,\ and\ conjugate$$

$$giving\ 3\ slowest\ decaying\ terms\ of$$

$$\begin{aligned} delta(t) =, &-.9420123470\, e^{(-.3181315050\, t)} \cos(1.337235701\, t) \\ &+ .2285130310\, e^{(-.3181315050\, t)} \sin(1.337235701\, t) \\ &- .03051313668\, e^{(-2.062277729\, t)} \cos(7.588631178\, t) \\ &- .01306036569\, e^{(-2.062277729\, t)} \sin(7.588631178\, t) \\ &- .009561746830\, e^{(-2.653191974\, t)} \cos(13.94920833\, t) \\ &- .003019967934\, e^{(-2.653191974\, t)} \sin(13.94920833\, t) \end{aligned}$$



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> finddelta(7.854);
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solution of d Delta(t)/dt = −lambda ∗ Delta(t − 1)

for lambda =, 7.854

Laplace transform's poles are at

$sr =, 1.185388725 + 2.087285406\, I,$ *and conjugate*

$sr =, .2300997353\, 10^{-5} + 7.853981927\, I,$ *and conjugate*

$sr =, -.5857050241 + 14.09563861\, I,$ *and conjugate*

giving 3 slowest decaying terms of

$$delta(t) =, -.5272326548\, e^{(1.185388725\, t)} \cos(2.087285406\, t) + 2.100236190\, e^{(1.185388725\, t)} \sin(2.087285406\, t) - .2505861086\, e^{(.2300997353\, 10^{-5}\, t)} \cos(7.853981927\, t) + .03190576110\, e^{(.2300997353\, 10^{-5}\, t)} \sin(7.853981927\, t) - .07895092850\, e^{(-.5857050241\, t)} \cos(14.09563861\, t) - .0009589118512\, e^{(-.5857050241\, t)} \sin(14.09563861\, t)$$