Spatial and Temporal Averaging in Combustion Chambers

F.E.C. Culick
California Institute of Technology

18 January, 2010
Combustion Workshop
IIT Chennai

Reference RTO 039

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

The General Context

(Munich)

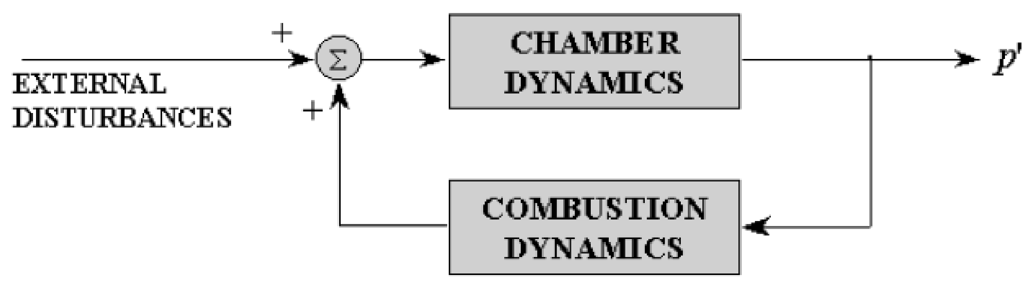


FIGURE 1.1. Schematic diagram of a combustion system as a feedback amplifier.

The Line of Combustion Instabilities

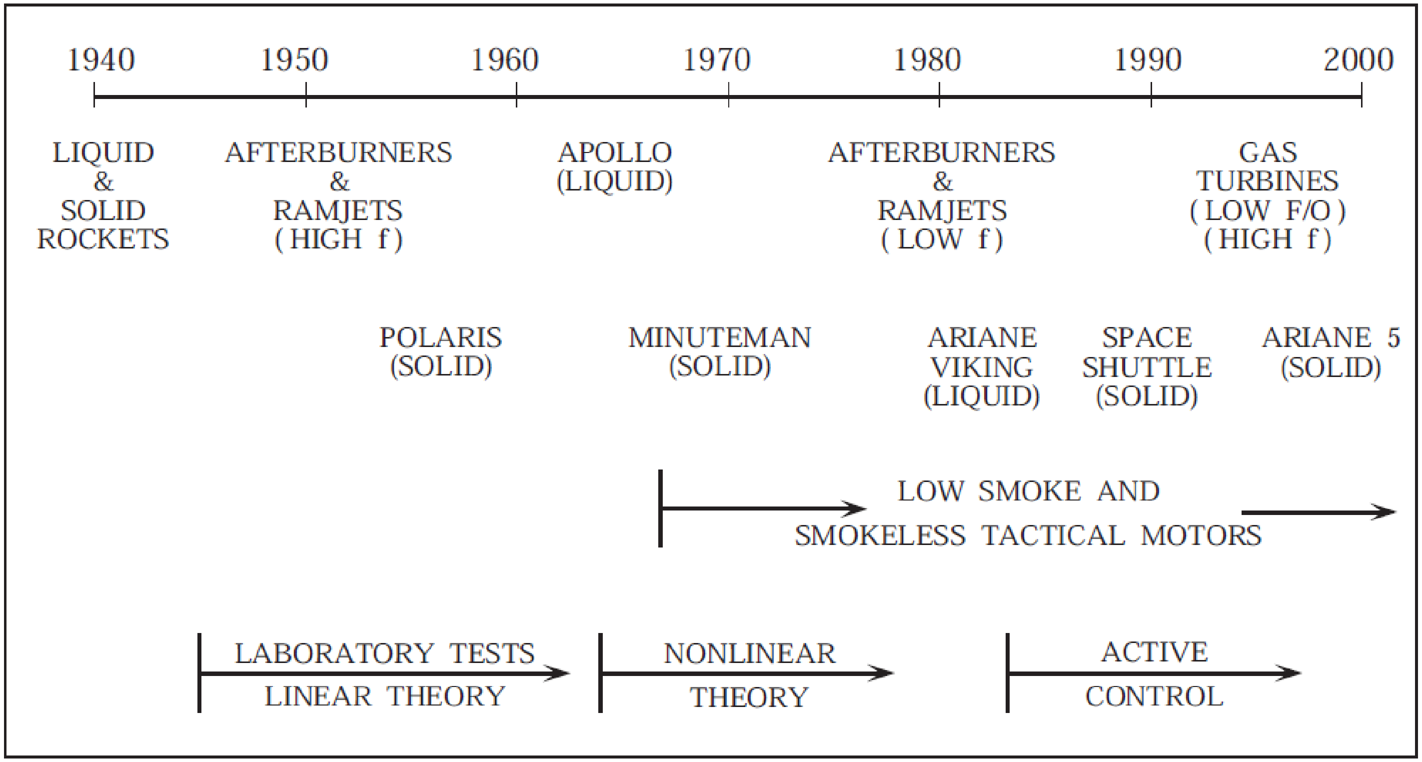


FIGURE 1.2. A chronology of combustion instabilities.

A Good Example

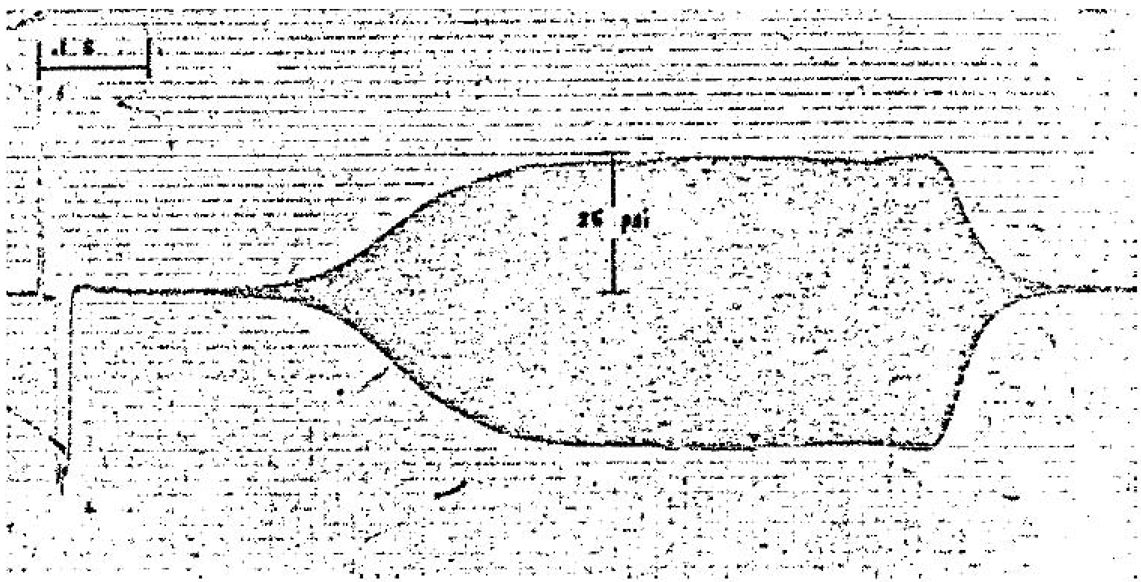


FIGURE 1.36. Exponential growth and development of a limit cycle out of a linearly unstable motion (Perry 1968).

One Approximation (Early)

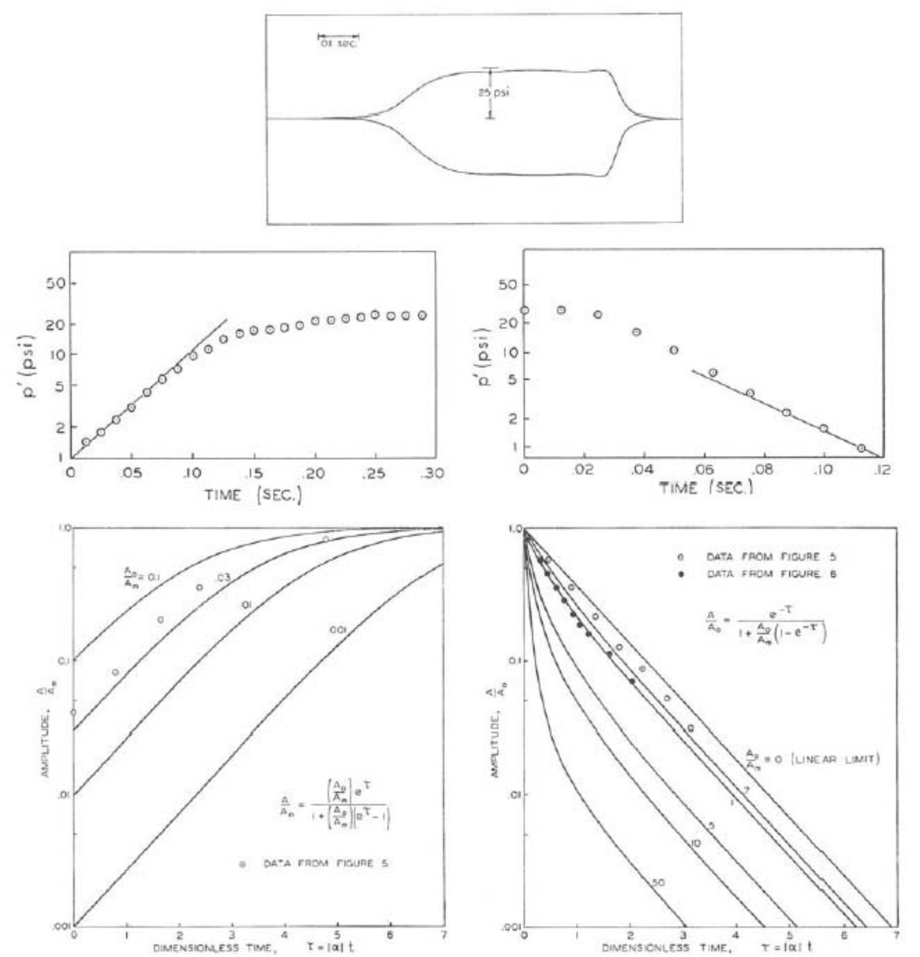


FIGURE 1.40. An example of fitting T-burner data with the model of a simple nonlinear oscillator (Culick 1971).

A Computational Result

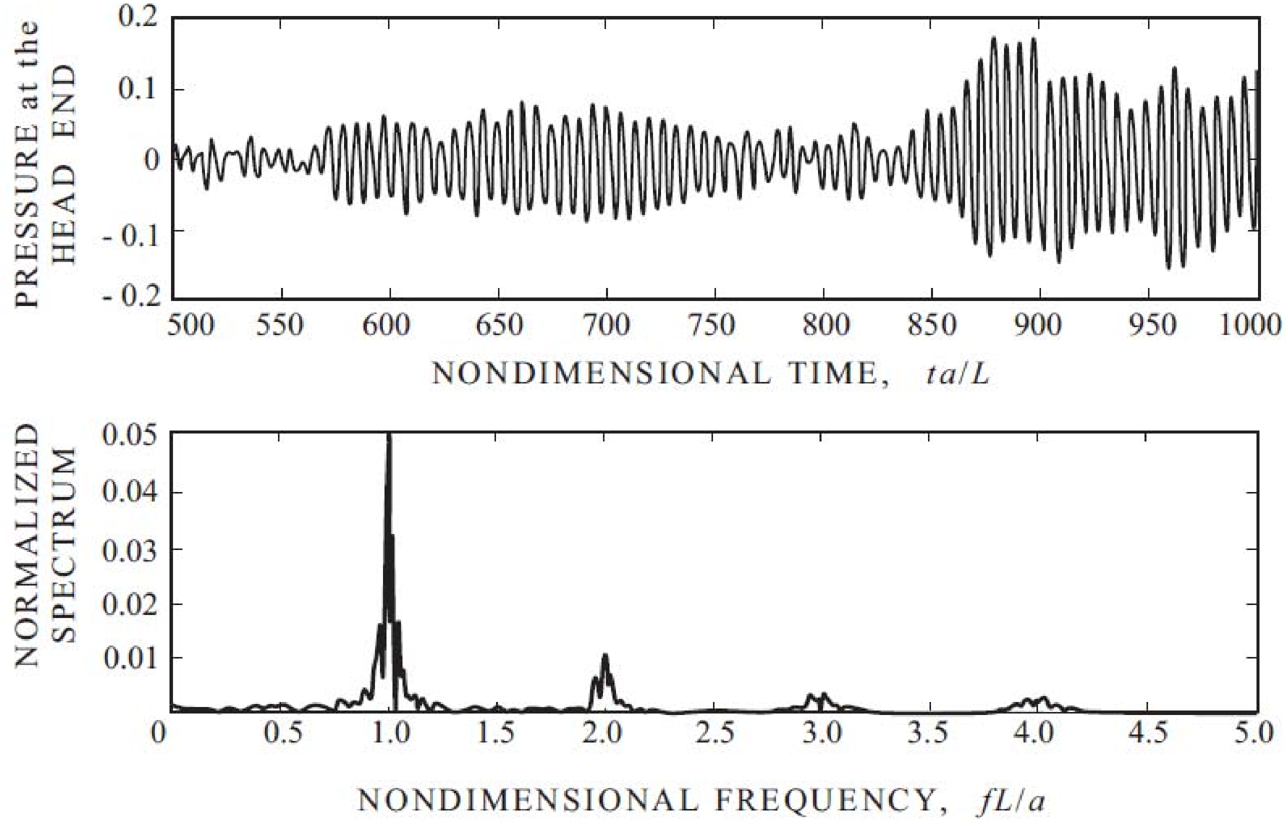


FIGURE 1.39. A computed limit cycle and its normalized spectrum executed by a single nonlinear acoustic mode in the presence of noise (Burnley and Culick 1996).

A Simple Laboratory Demonstration

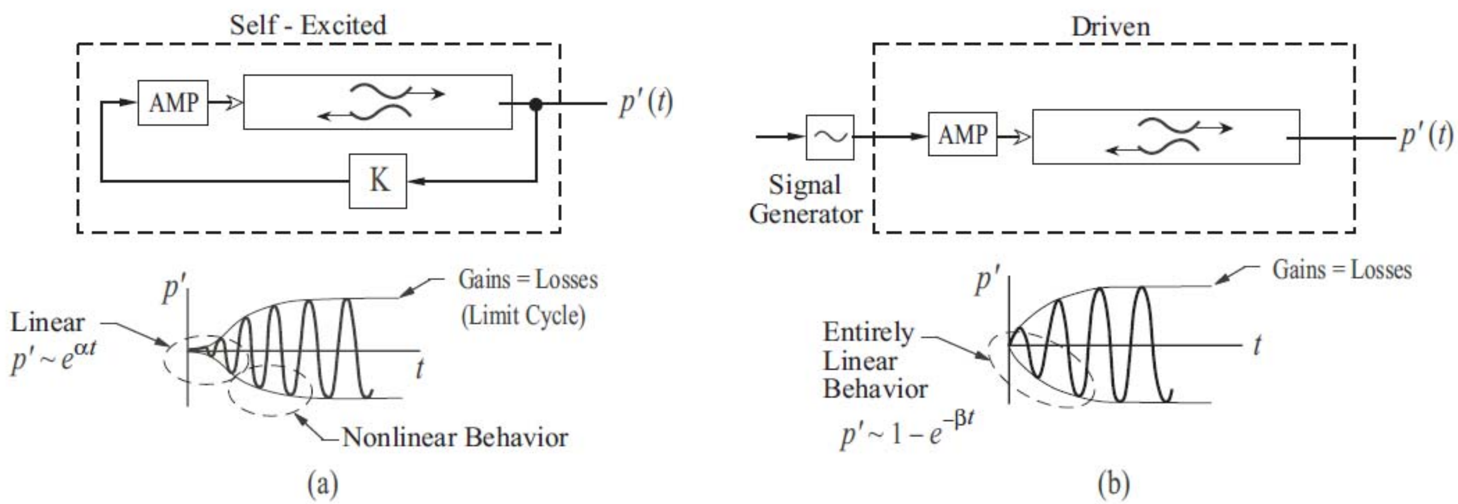


FIGURE 1.34. Transient behavior of (a) self-excited linearly unstable motions; (b) forced motions.

The Grand Picture

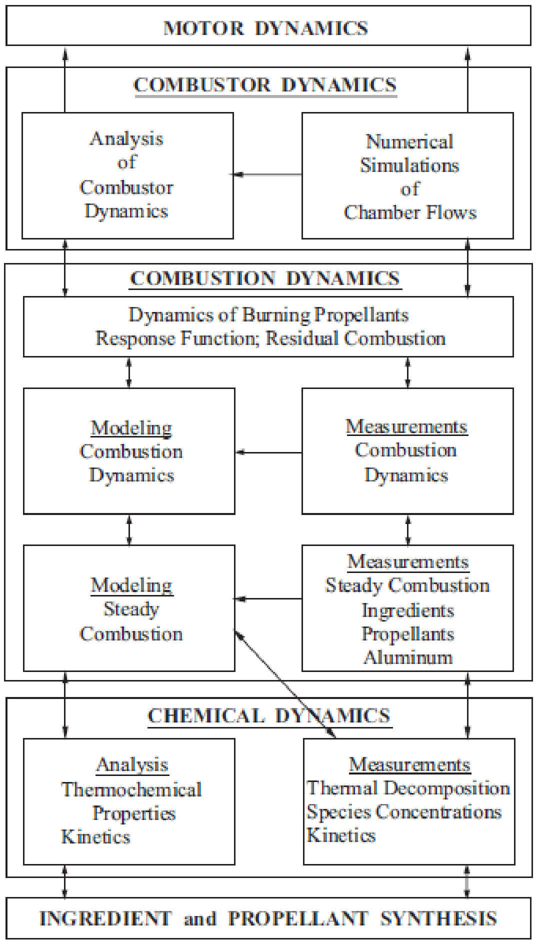


FIGURE 2.11. A view of the areas of research and their connections in solid propellant rockets (Culick 2000).

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for $p^{'}$ and $\mathbf{M}^{'}$, Linear Motions

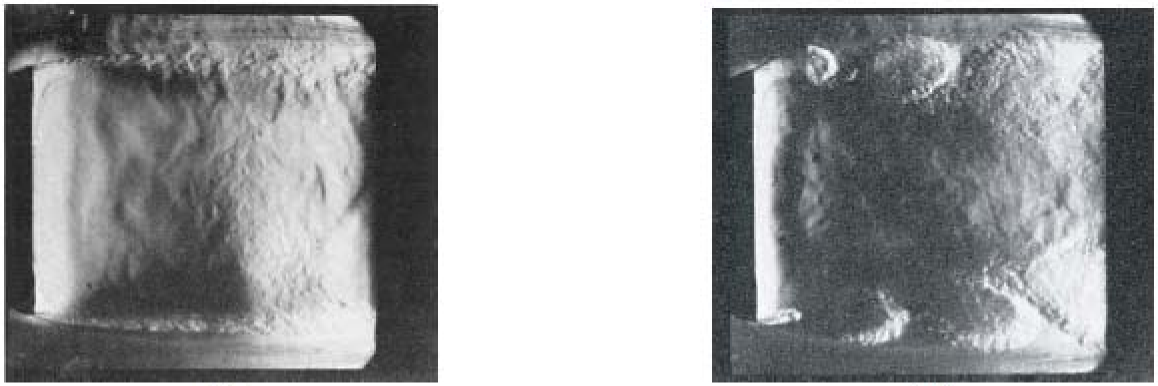
VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Vortex Shedding From a Bluff Flame Holder



(a) (b)

FIGURE 2.39. Flow past a bluff body flameholder under two conditions of flow at approximately the same speed. (a) low equivalence ratio, $\phi \lesssim 0.75$, no oscillations; (b) high equivalence ratio $\phi \gtrsim 0.90$, acoustic oscillations in the channel (Rogers and Marble 1956).

Simple Vortex Shedding/Driving

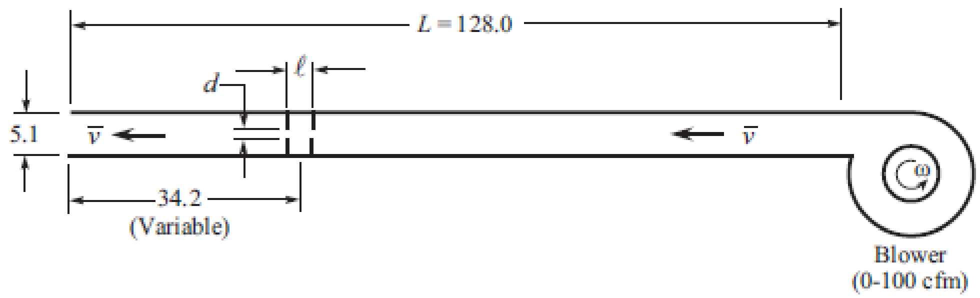


FIGURE 2.29. Sketch of an apparatus for demonstrating the excitation of acoustic modes by vortex shedding at a pair of annuli. All dimensions in centimeters (adapted from Culick and Magiawala 1979).

Vortex Shedding & Acoustics

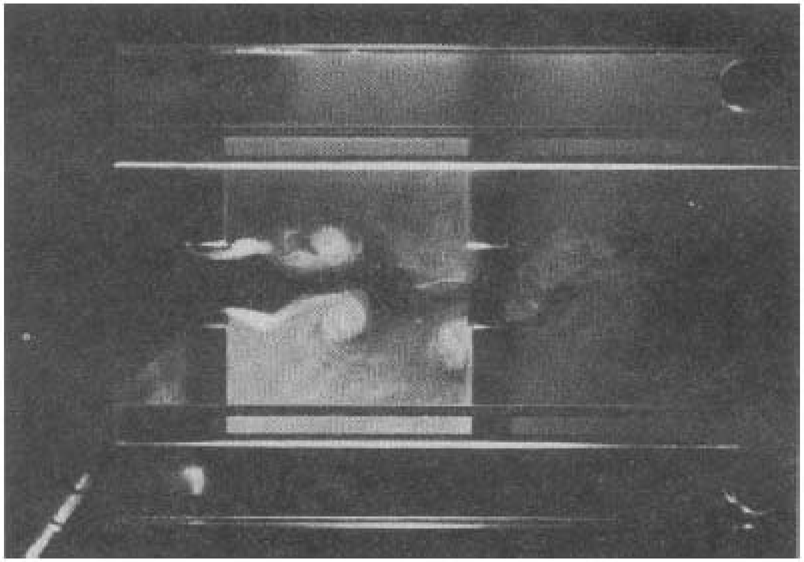


FIGURE 2.31. Typical flow between the baffles when a pure tone is generated (Nomoto and Culick 1982).

Acoustic Modes and Vortex Shedding

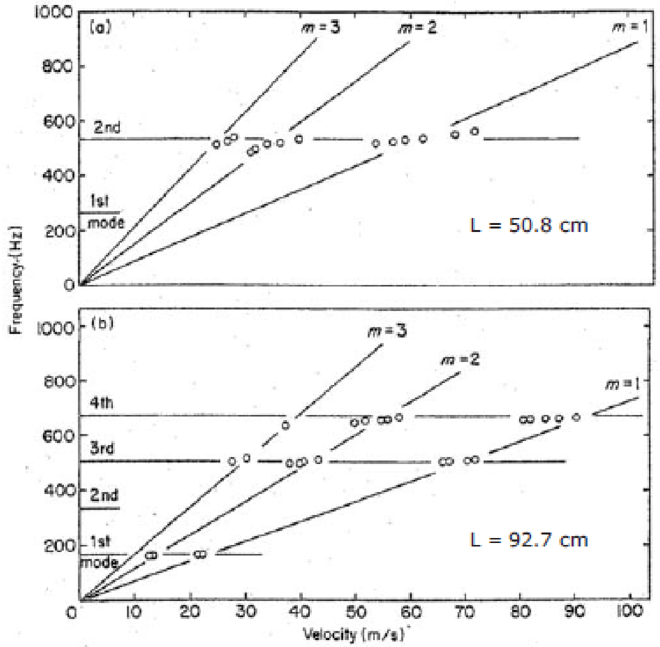


FIGURE 2.32. Experimental results for the excitation of acoustic modes by vortex shedding (Nomoto and Culick 1982). Open circles identify conditions when significant oscillations were observed. The length of the chambers from inlet to exhaust.

Summary of Mechanisms for CI in Solid Rockets

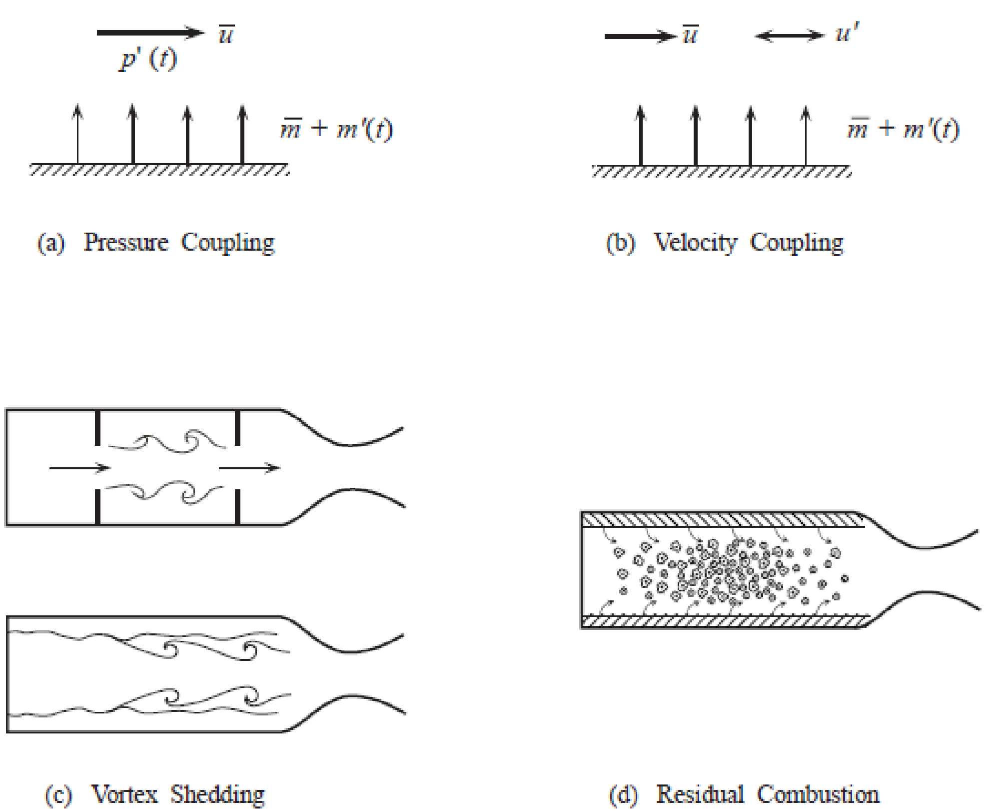


FIGURE 2.1. The four possible mechanisms for combustion instabilities in solid propellant rockets.

Three Sorts of Vortex Shedding in Solid Rockets

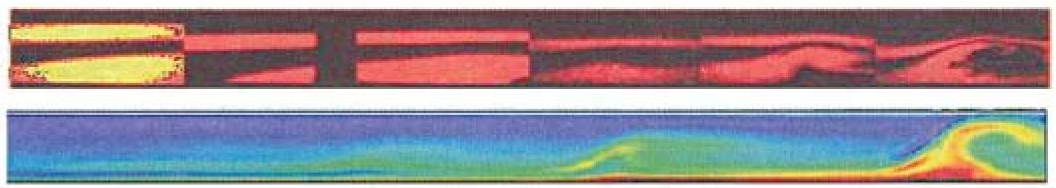


FIGURE 6.27. Upper image: flow visualization of parietal vortex shedding, using PLIF with acetone; Lower image: result of numerical calculations (Avalon *et al.* 2001).

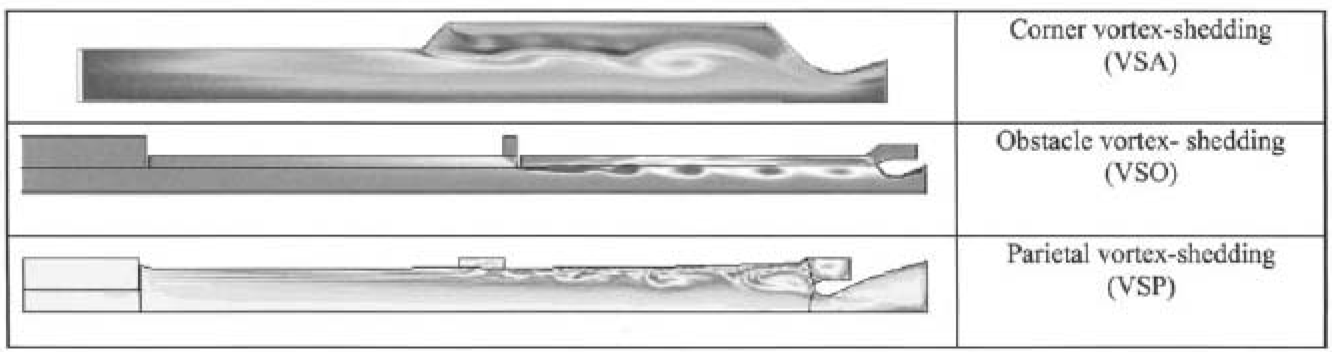


FIGURE 6.28. The three kinds of vortex shedding (Fabignon *et al.* 2003).

A Result of a Sub-Scale Test

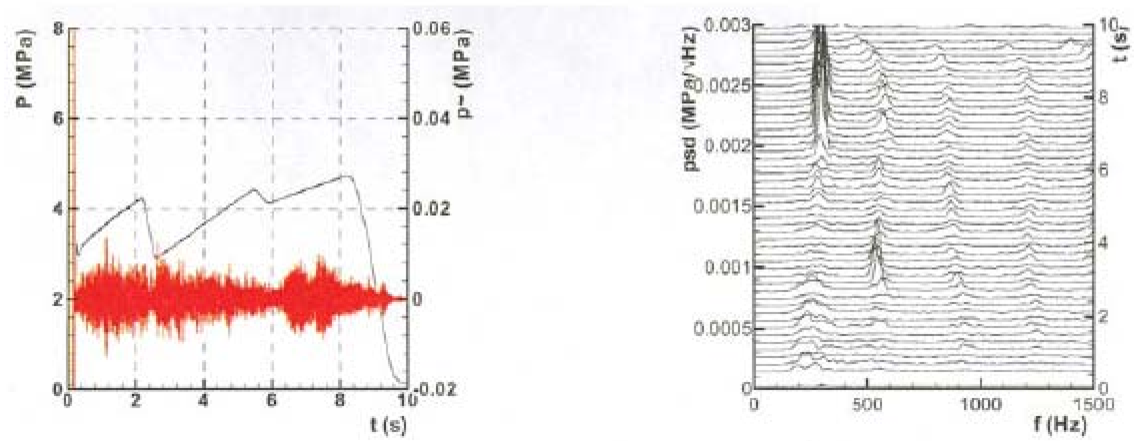


FIGURE 6.30. Experimental results for a test in the LP6 configuration showing parietal vortex shedding (Prévost et al. 2005).

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\mathbf{M}'$, Linear Motions

VI Time Averaging

VII Rayleigh’s Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Basis of Approximate Analysis

- ‘Complete’ equations of motion
- Express as mean plus unsteady flows
- Two ‘small’ expansion parameters: $\varepsilon \sim p'; \mu \sim \bar{u}$
- Expand equations to $O\{\mu\}$ and $O\{(\varepsilon^2)\}$ or $O\{(\varepsilon^3)\}$
- Expand dependent variables (p',u') in normal modes **(N.B. boundary conditions are for cases of <u>no</u> flow and <u>no</u> combustion)**

Basis of Approximate Analysis (cont'd)

- Form perturbed wave equation for p'
- Expand $p'(\mathbf{r},t)$ in modes with time-varying amplitudes η_m
- Oscillator equations for η_m are inhomogeneous, coupled and nonlinear
  - At this point, no physical processes are ignored
- N.B. the dependent variables p', $\mathbf{n}'$, satisfy correct (inhomogeneous) B.C. but the expansion basis functions do <u>not.</u>

Equations With No Approximations

$$\left[\frac{\bar{D}\bar{\rho}}{Dt}+\bar{\rho}\nabla\cdot\bar{\mathbf{M}}+\bar{\mathbf{M}}\cdot\nabla\bar{\rho}-\bar{\mathcal{W}}\right]+\left[\frac{\partial\rho'}{\partial t}+\bar{\rho}\nabla\cdot\mathbf{M}'\right]$$
$$+\left[\bar{\mathbf{M}}\cdot\nabla\rho'+\rho'\nabla\cdot\bar{\mathbf{M}}+\mathbf{M}'\cdot\nabla\bar{\rho}+\nabla\cdot(\rho'\mathbf{M}')\right]-\mathcal{W}'=0 \tag{3.15}$$

$$\left[\bar{\rho}\frac{\bar{D}\mathbf{M}'}{Dt}+\nabla\bar{p}-\bar{\mathfrak{F}}\right]+\left[\bar{\rho}\frac{\partial\bar{\mathbf{M}}}{\partial t}+\nabla p'\right]+\left[\bar{\rho}\left(\bar{\mathbf{M}}\cdot\nabla\mathbf{M}'+\mathbf{M}'\cdot\nabla\bar{\mathbf{M}}\right)+\rho'\frac{\bar{D}\bar{\mathbf{M}}}{Dt}\right]$$
$$+\left[\rho'\frac{\partial\mathbf{M}'}{\partial t}+\bar{\rho}\mathbf{M}'\cdot\nabla\mathbf{M}'+\rho'\left(\bar{\mathbf{M}}\cdot\nabla\mathbf{M}'+\mathbf{M}'\cdot\nabla\bar{\mathbf{M}}\right)\right]+\left[\rho'\mathbf{M}'\cdot\nabla\mathbf{M}'\right]-\mathfrak{F}'=0 \tag{3.16}$$

$$\left[\bar{\rho}C_v\frac{\bar{D}\bar{T}}{Dt}+\bar{p}\nabla\cdot\bar{\mathbf{M}}-\bar{\mathcal{Q}}\right]+C_v\left[\bar{\rho}\frac{\partial T'}{\partial t}+\bar{p}\nabla\cdot\mathbf{M}'\right]+\left[\bar{\rho}C_v\left(\bar{\mathbf{M}}\cdot\nabla T'+\mathbf{M}'\cdot\nabla\bar{T}\right)+C_v\rho'\frac{\bar{D}\bar{T}}{Dt}+p'\nabla\cdot\bar{\mathbf{M}}\right]$$
$$+\left[C_v\bar{\rho}\frac{\partial T'}{\partial t}+C_v\rho'\left(\bar{\mathbf{M}}\cdot\nabla T'+\mathbf{M}'\cdot\nabla\bar{T}\right)+C_v\rho'\mathbf{M}'\cdot\nabla T'+p'\nabla\cdot\mathbf{M}'\right]+\left[C_v\bar{\rho}\mathbf{M}'\cdot\nabla T'\right]-\mathcal{Q}'=0 \tag{3.17}$$

Equations for Mean Flow (1)

$$\bar{\mathbf{M}} \cdot \nabla\bar{\rho} + \bar{\rho}\nabla \cdot \bar{\mathbf{M}} = \bar{\mathcal{W}} \tag{3.28}$$

$$\bar{\rho}\bar{\mathbf{M}} \cdot \nabla\bar{\mathbf{M}} + \nabla\bar{p} = \bar{\mathfrak{F}} \tag{3.29}$$

$$\bar{\rho}C_v\bar{\mathbf{M}} \cdot \nabla\bar{T} + \bar{p}\nabla \cdot \bar{\mathbf{M}} = \bar{\Omega} \tag{3.30}$$

$$\bar{\mathbf{M}} \cdot \nabla\bar{p} + \gamma\bar{p}\nabla \cdot \bar{\mathbf{M}} = \bar{\mathcal{P}} \tag{3.31}$$

$$\bar{\rho}\bar{T}\bar{\mathbf{M}} \cdot \nabla\bar{s} = \bar{\mathcal{S}} \tag{3.32}$$

$$\bar{p} = R\bar{\rho}\bar{T} \tag{3.33}$$

Some Definitions of Special Symbols

$$
\begin{aligned}
\{[\rho]\}_1 &= \bar{\mathbf{M}} \cdot \nabla \rho' + \mathbf{M}' \cdot \nabla \bar{\rho} + \rho' \nabla \cdot \bar{\mathbf{M}} \\
\{\rho\}_2 &= \mathbf{M}' \cdot \nabla \rho' + \rho' \nabla \cdot \mathbf{M}' \\
\{[M]\}_1 &= \bar{\rho} \left(\bar{\mathbf{M}} \cdot \nabla \mathbf{M}' + \mathbf{M}' \cdot \nabla \bar{\mathbf{M}} \right) + \rho' \frac{\bar{D} \bar{\mathbf{M}}}{Dt} \\
\{M\}_2 &= \rho' \frac{\partial \mathbf{M}'}{\partial t} + \rho' (\bar{\mathbf{M}} \cdot \nabla \mathbf{M}' + \mathbf{M}' \cdot \nabla \bar{\mathbf{M}}) + \bar{\rho} \mathbf{M}' \cdot \nabla \mathbf{M}' \\
\{[M]\}_2 &= \rho' \left(\bar{\mathbf{M}} \cdot \nabla \mathbf{M}' + \mathbf{M}' \cdot \nabla \bar{\mathbf{M}} \right) \\
\{M\}_3 &= \rho' \mathbf{M}' \cdot \nabla \mathbf{M}'
\end{aligned}
$$

$$
\begin{aligned}
\{[p]\}_1 &= \bar{\mathbf{M}} \cdot \nabla p' + \gamma p' \nabla \cdot \bar{\mathbf{M}} + \mathbf{M}' \cdot \nabla \bar{p} \\
\{p\}_2 &= \mathbf{M}' \cdot \nabla p' + \gamma p' \nabla \cdot \mathbf{M}' \\
\{[T]\}_1 &= \bar{\rho} C_v \left(\bar{\mathbf{M}} \cdot \nabla T' + \mathbf{M}' \cdot \nabla \bar{T} \right) + \\
&\quad + C_v \rho' \frac{\bar{D} \bar{T}}{Dt} + p' \nabla \cdot \bar{\mathbf{M}} \\
\{T\}_2 &= \rho' C_v \frac{\partial T'}{\partial t} + \bar{\rho} C_v \mathbf{M}' \cdot \nabla T' + p' \nabla \cdot \mathbf{M}' \\
\{[T]\}_2 &= \rho' C_v \left(\bar{\mathbf{M}} \cdot \nabla T' + \mathbf{M}' \cdot \nabla \bar{T} \right) \\
\{T\}_3 &= \rho' C_v \mathbf{M}' \cdot \nabla T'
\end{aligned}
$$

Equations for Mean Flow (2)

$$\frac{\bar{D}\bar{\rho}}{Dt} + \bar{\rho}\nabla \cdot \bar{\mathbf{M}} = \bar{\mathcal{W}} - \overline{\{[\rho]\}_1} - \overline{\{\rho\}_2} + \overline{\mathcal{W}'} \qquad (3.34)$$

$$\bar{\rho}\frac{\bar{D}\bar{\mathbf{M}}}{Dt} + \nabla\bar{p} = \bar{\mathfrak{F}} - \overline{\{[\mathbf{M}]\}_1} - \overline{\{\mathbf{M}\}_2} - \overline{\{\mathbf{M}\}_3} - \overline{\{[\mathbf{M}]\}_2} + \overline{\mathfrak{F}'} \qquad (3.35)$$

$$\bar{\rho}C_v\frac{\bar{D}\bar{T}}{Dt} + \bar{p}\nabla \cdot \bar{\mathbf{M}} = \bar{\mathcal{Q}} - \overline{\{[T]\}_1} - \overline{\{T\}_2} - \overline{\{T\}_3} + \bar{\mathcal{Q}}' \qquad (3.36)$$

$$\frac{\bar{D}\bar{p}}{Dt} + \gamma\bar{p}\nabla \cdot \bar{\mathbf{M}} = \bar{\mathcal{P}} - \overline{\{p\}_1} - \overline{\{p\}_2} + \bar{\mathcal{P}}' \qquad (3.37)$$

$$\bar{\rho}\bar{T}\frac{\bar{D}\bar{s}}{Dt} = \bar{\mathcal{S}} - \overline{\{s\}_1} - \overline{\{s\}_2} - \overline{\{s\}_3} - \overline{\{[s]\}_2} - \overline{\{s\}_4} + \bar{\mathcal{S}}' \qquad (3.38)$$

$$\bar{p} = \mathbf{R}\bar{\rho}\bar{T} - \overline{\{\rho T\}_1} - \overline{\{\rho T\}_2} \qquad (3.39)$$

Equations With Terms Collected by Order

$$\left[\frac{\bar{D}\bar{\rho}}{Dt} + \bar{\rho}\nabla\cdot\bar{\mathbf{M}} - \bar{\mathcal{W}}\right] + \left(\frac{\partial\rho'}{\partial t} + \bar{\rho}\nabla\cdot\mathbf{M}'\right) + \{[\rho]\}_1 + \{\rho\}_2 - \mathcal{W}' = 0 \qquad (3.22)$$

$$\left[\bar{\rho}\frac{\bar{D}\bar{\mathbf{M}}}{Dt} + \nabla\bar{p} - \bar{\mathcal{F}}\right] + \left(\bar{\rho}\frac{\partial\mathbf{M}'}{\partial t} + \nabla p'\right) + \{[\mathbf{M}]\}_1 + \{\mathbf{M}\}_2 + \{\mathbf{M}\}_3 + \{[\mathbf{M}]\}_2 - \mathcal{F}' = 0 \qquad (3.23)$$

$$\left[\bar{\rho}C_v\frac{\bar{D}\bar{T}}{Dt} + \bar{p}\nabla\cdot\bar{\mathbf{M}} - \bar{\mathcal{Q}}\right] + C_v\left(\bar{\rho}\frac{\partial T'}{\partial t} + \bar{p}\nabla\cdot M'\right) + \{[T]\}_1 + \{T\}_2 + \{T\}_3 + \{[T]\}_2 - \mathcal{Q}' = 0 \qquad (3.24)$$

$$\left[\frac{\bar{D}\bar{p}}{Dt} + \gamma\bar{p}\nabla\cdot\bar{\mathbf{M}} - \bar{\mathcal{P}}\right] + \left(\bar{\rho}C_v\frac{\partial P'}{\partial t} + \bar{p}\nabla\cdot M'\right) + \{[p]\}_1 + \{p\}_2 - \mathcal{P}' = 0 \qquad (3.25)$$

$$\left[\bar{\rho}\bar{T}\frac{\bar{D}\bar{s}}{Dt} - \bar{\mathcal{S}}\right] + \left(\bar{\rho}\bar{T}\frac{\partial s'}{\partial t}\right) + \{[s]\}_1 + \{s\}_2 + \{s\}_3 + \{[s]\}_2 + \{s\}_4 - \mathcal{S}' = 0 \qquad (3.26)$$

$$\left[\bar{p} - R\bar{\rho}\bar{T}\right] + \{p - R\rho T\}_1 + \{R\rho T\}_2 = 0 \qquad (3.27)$$

Equations for Fluctuations to Third Order

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\nabla \cdot \mathbf{M}' = -\{[\rho]\}_1 - \{\rho\}_2 + \mathcal{W}' \tag{3.40}$$

$$\bar{\rho}\frac{\partial \mathbf{M}'}{\partial t} + \nabla p' = -\{[\mathbf{M}]\}_1 - \{\mathbf{M}\}_2 - \{\mathbf{M}\}_3 - \{[\mathbf{M}]\}_2 + \mathcal{F}' \tag{3.41}$$

$$\bar{\rho}C_v\frac{\partial T'}{\partial t} + \bar{p}\nabla \cdot \mathbf{M}' = -\{[T]\}_1 - \{T\}_2 - \{T\}_3 - [\{T\}_2] + \Omega' \tag{3.42}$$

$$\frac{\partial p'}{\partial t} + \gamma\bar{p}\nabla \cdot \mathbf{M}' = -\{[p]\}_1 - \{p\}_2 + \mathcal{P}' \tag{3.43}$$

$$\bar{\rho}\bar{T}\frac{\partial s'}{\partial t} = -\{[s]\}_1 - \{s\}_2 - \{[s]\}_2 - \{s\}_3 - \{s\}_4 + \mathcal{S}' \tag{3.44}$$

Equations for Linear Stability

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\nabla \cdot \mathbf{M}' = -\{[\rho]\}_1 + \mathcal{W}'$$

$$\bar{\rho}\frac{\partial \mathbf{M}'}{\partial t} + \nabla p' = -\{[\mathbf{M}]\}_1 + \boldsymbol{\mathcal{F}}'$$

$$\bar{\rho}C_v\frac{\partial T'}{\partial t} + \bar{p}\nabla \cdot \mathbf{M}' = -\{[T]\}_1 + \mathcal{Q}' \qquad (3.47)\text{ a-e}$$

$$\frac{\partial p'}{\partial t} + \gamma\bar{p}\nabla \cdot \mathbf{M}' = -\{[p]\}_1 + \mathcal{P}'$$

$$\bar{\rho}\bar{T}\frac{\partial s'}{\partial t} = -\{[s]\}_1 + \mathcal{S}'$$

Second Order Acoustics

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\nabla \cdot \mathbf{M}' = -(\{[\rho]\}_1 + \{\rho\}_2) + \mathcal{W}'$$

$$\bar{\rho}\frac{\partial \mathbf{M}'}{\partial t} + \nabla p' = -(\{[\mathbf{M}]\}_1 + \{\mathbf{M}\}_2) + \mathcal{F}'$$

$$\bar{\rho}C_v\frac{\partial T'}{\partial t} + \bar{p}\nabla \cdot \mathbf{M}' = -(\{[T]\}_1 + \{T\}_2) + \mathcal{Q}' \qquad (3.48)\text{ a-e}$$

$$\frac{\partial p'}{\partial t} + \gamma\bar{p}\nabla \cdot \mathbf{M}' = -(\{[p]\}_1 + \{p\}_2) + \mathcal{P}'$$

$$\bar{\rho}\bar{T}\frac{\partial s'}{\partial t} = -(\{[s]\}_1 + \{s\}_2) + \mathcal{S}'$$

Nonlinear Wave Equation (Third Order)

$$\nabla^2 p' - \frac{1}{\bar{a}^2}\frac{\partial^2 p'}{\partial t^2} = h$$
$$\hat{\mathbf{n}} \cdot \nabla p' = -f \qquad (3.57)\text{a,b}$$

$$h = \left[-\bar{\rho}\nabla \cdot \frac{1}{\bar{\rho}}\{[\mathrm{M}]\}_1 + \frac{1}{\bar{a}^2}\frac{\partial\{[p]\}_1}{\partial t} + \frac{1}{\bar{\rho}}\nabla\bar{\rho}\cdot\nabla p'\right] + \left[-\bar{\rho}\nabla\cdot\frac{1}{\bar{\rho}}\{\mathrm{M}\}_2 + \frac{1}{\bar{a}^2}\frac{\partial\{p\}_2}{\partial t}\right]$$
$$- \left[\bar{\rho}\nabla\cdot\frac{1}{\bar{\rho}}\{\mathrm{M}\}_3\right] - \left[\bar{\rho}\nabla\cdot\frac{1}{\bar{\rho}}\{[\mathrm{M}]\}_2\right] + \left[+\bar{\rho}\nabla\cdot\frac{1}{\bar{\rho}}\mathcal{F}' - \frac{1}{\bar{a}^2}\frac{\partial \mathcal{P}'}{\partial t}\right] \qquad (3.58)$$

$$f = \bar{\rho}\frac{\partial \mathrm{M}'}{\partial t}\cdot\hat{\mathbf{n}} + \hat{\mathbf{n}}\cdot\left[\{[\mathrm{M}]\}_1 + \{\mathrm{M}\}_2 + \{\mathrm{M}\}_3 + \{[\mathrm{M}]\}_2\right] - \mathcal{F}'\cdot\hat{\mathbf{n}} \qquad (3.59)$$

Linear Equations, Steady Waves

$$\nabla^2 p' - \frac{1}{\bar{a}^2}\frac{\partial^2 p'}{\partial t^2} = h$$
$$\hat{\mathbf{n}} \cdot \nabla p' = -f \qquad (4.1)\text{a,b}$$

$$p' = \hat{p} e^{-i\bar{a}kt} \qquad (4.2)$$

$$k = \frac{1}{\bar{a}}(\omega + i\alpha) \qquad (4.3)$$

$$\nabla^2 \hat{p} + k^2 \hat{p} = \kappa \hat{h}$$
$$\hat{\mathbf{n}} \cdot \nabla \hat{p} = -\kappa \hat{f} \qquad (4.4)\text{a,b}$$

First Order Solution, by Iteration

$$
\begin{aligned}
(k^2)^{(2)} &= k_N^2 + \frac{\kappa}{E_N^2} \iiint\limits_V \psi_N H(\mathbf{r}_0)(\psi_N + \kappa\sigma_N)dV_0 \\
&= (k^2)^{(1)} + \kappa^2 \frac{\kappa}{E_N^2} \iiint\limits_V \psi_N H(\mathbf{r}_0)\sigma_N dV_0
\end{aligned}
\qquad (4.25)
$$

First Order Results for Linear Harmonic Motions

$$\hat{p}^{(1)}(\mathbf{r}) = \psi_N(\mathbf{r}) + \sum_{n=0}^{\infty}{}' \frac{\psi_n(\mathbf{r})}{E_n^2\left(k^2 - k_n^2\right)} \left\{ \iiint_V \psi_n(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_n(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.82}$$

$$i\bar{\rho}\bar{a}k\mathbf{M}'^{(1)} = -\nabla\hat{p}^{(1)} - \varepsilon\mu\{[\hat{\mathbf{M}}]\}_1 + \varepsilon\hat{\mathfrak{F}}_{10} + \varepsilon\mu\hat{\mathfrak{F}}_{11} \tag{4.85}$$

$$k^2 = k_N^2 + \frac{1}{E_N^2} \left\{ \iiint_V \psi_N(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_N(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.19}$$

(4.85) shows that due to mean flow interactions and other contributions, the system of equations is always “non-normal” except in <u>very</u> special cases.

Two Results of One-Dimensional Representation

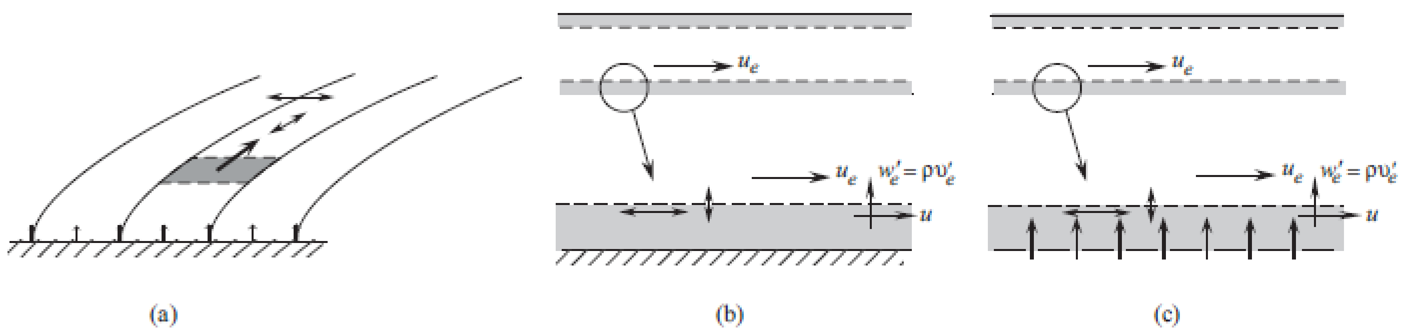


FIGURE 2.28. Sketches illustrating two primary processes involved in the generation of vorticity. (a) flow-turning; (b) 'pumping action': oscillatory motion parallel to the boundary, in the boundary layer, induces oscillatory motion normal to an impermeable wall; (c) similar to (b) with mean flow through the wall.

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Conservation Equations for One-Dimensional Flows

Mass

$$\frac{D\rho}{Dt} = -\rho \frac{1}{S_c} \frac{\partial}{\partial x} (S_c u) + (\mathcal{W}_1 + \mathcal{W}_{1s}) \tag{B.2}$$

Momentum

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + (\mathcal{F}_1 + \mathcal{F}_{1s}) \tag{B.3}$$

Energy

$$\rho C_v \frac{DT}{Dt} = -p \frac{1}{S_c} \frac{\partial}{\partial x} (S_c u) + (\mathcal{Q}_1 + \mathcal{Q}_{1s}) \tag{B.4}$$

Pressure

$$\frac{Dp}{Dt} = -\gamma p \frac{1}{S_c} \frac{\partial}{\partial x} (S_c u) + (\mathcal{P}_1 + \mathcal{P}_{1s}) \tag{B.5}$$

Entropy

$$\frac{Ds}{Dt} = \frac{1}{T} (\mathcal{S}_1 + \mathcal{S}_{1s}) \tag{B.6}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \tag{B.7}$$

Source Terms in One-Dimensional Flows

$$\mathcal{W}_1 = -\frac{1}{S_c}\frac{\partial}{\partial x}(S_c \rho_l \delta u_l) + w_e \tag{B.8}$$

$$\mathcal{F}_1 = \frac{\partial \tau_v}{\partial x} + m_e + m_D - \sigma_e - \delta u_l w_g^{(l)} - \rho_l \frac{D\delta u_l}{Dt} \tag{B.9}$$

$$\begin{aligned} \mathcal{Q}_1 = {} & \tau_v \frac{\partial u}{\partial x} - \frac{\partial q}{\partial x} + Qw + Q_e - (\epsilon_{og} w_g + \epsilon_{ol} w_l) \\ & + \chi_0 + \Sigma h_i \frac{\partial}{\partial x}(\rho_g V_i Y_{gi}) - u(m_e + m_D - \sigma_e) \\ & + (u\delta u_l) w_g^{(l)} + \delta Q_l + \delta u_l F_l - u(\mathcal{F}_1 - F_l) \end{aligned} \tag{B.10}$$

$$\mathcal{P}_1 = \frac{R}{C_v}\mathcal{Q}_1 + RT\left[\mathcal{W}_1 - \frac{\partial}{\partial x}(\rho_l \delta u_l)\right] \tag{B.11}$$

$$\mathcal{S}_1 = \mathcal{Q}_1 - \frac{p}{\rho^2}\mathcal{W}_1 \tag{B.12}$$

Sources Due To Flow Through Lateral Boundaries

$$\mathcal{W}_{1s} = \frac{1}{S_c}\int m_s dq = \frac{1}{S_c}\int [m_{sg} + m_{sl}]\, dq \tag{B.13}$$

$$\mathcal{F}_{1s} = \frac{1}{S_c}\int [(u_s - u)m_{sg} + (u_{ls} - u_l)m_{sl}]\, dq \tag{B.14}$$

$$\mathcal{Q}_{1s} = \frac{1}{S_c}\int [(h_{0s} - e_0)m_{sg} + (e_{l0s} - e_{l0})m_{sl}]\, dq \tag{B.15}$$

$$\mathcal{P}_{1s} = \frac{R}{C_v}\mathcal{Q}_{1s} + RT_s\mathcal{W}_{1s} = \frac{R}{C_v}\frac{1}{S_c}\int [(h_{0s} - e_0)m_{sg} + (e_{l0s} - e_{l0})m_{sl} + C_vTm_{sg}]\, dq \tag{B.16}$$

$$\mathcal{S}_{1s} = \frac{1}{\rho}\mathcal{Q}_1 - \frac{p}{\rho}\mathcal{W}_{1s} \tag{B.17}$$

A useful form for $\mathcal{P}_{1s}$ is

$$\mathcal{P}_{1s} = \frac{1}{S_c}\int (a^2 + \gamma R\Delta T_s)m_{sg}dq + \frac{R}{C_v}\frac{1}{S_c}\int (e_{l0s} - e_{l0})m_{sl}\, dq \tag{B.18}$$

where $\Delta T_s = T_s - T$. Terms containing $u_s^2 m_{sg}$ or $u^2 m_{sg}$ are of higher order and therefore are negligible.

Sources First Order In Fluctuations and M

$$\mathcal{W}'_{1s} = \frac{1}{S_c}\int m'_s dq \tag{B.19}$$

$$\begin{aligned}\mathcal{F}'_{1s} = \frac{1}{S_c}&\left\{(\bar{u}_s - \bar{u})\int m'_{sg}dq + (\bar{u}_{ls} - \bar{u})\int m'_{sl}dq\right\}\\ &+\frac{1}{S_c}\left\{(u'_s - u')\int \bar{m}_{sg}dq + (u'_{ls} - u'_l)\int \bar{m}_{sl}dq\right\}\end{aligned} \tag{B.20}$$

$$\begin{aligned}\mathcal{Q}'_{1s} = \frac{1}{S_c}&\left\{(\bar{h}_{0s} - \bar{e}_0)\int m'_{sg}dq + (\bar{e}_{l0s} - \bar{e}_{l0})\int m'_{sl}dq + C_v\bar{T}\int m'_{sg}dq\right\}\\ &+\frac{1}{S_c}\left\{(h'_{0s} - e'_0)\int \bar{m}_{sg}dq + (e'_{l0s} - e'_{l0})\int \bar{m}_{sl} + C_vT'\int \bar{m}_{sg}dq\right\}\end{aligned} \tag{B.21}$$

$$\begin{aligned}\mathcal{P}'_{1s} = \frac{R}{C_v}\frac{1}{S_c}&\left\{(\bar{h}_{0s} - \bar{e}_0)\int m'_{sg}dq + (\bar{e}_{l0s} - \bar{e}_{l0})\int m'_{sl}dq + C_v\bar{T}\int m'_{sg}dq\right\}\\ &+\frac{R}{C_v}\frac{1}{S_c}\left\{(h'_{0s} - e'_0)\int \bar{m}_{sg}dq + (e'_{l0s} - e'_{l0})\int \bar{m}_{sl} + C_vT'\int \bar{m}_{sg}dq\right\}\end{aligned} \tag{B.22}$$

$$\mathcal{S}'_{1s} = \frac{1}{p}\mathcal{Q}'_{1s} - \frac{p}{\rho}\mathcal{W}'_{1s} \tag{B.23}$$

Nonlinear Wave Equation and Boundary Condition

$$\frac{1}{S_c}\frac{\partial}{\partial x}\left(S_c\frac{\partial p'}{\partial x}\right) - \frac{1}{\bar{a}^2}\frac{\partial^2 p'}{\partial t^2} = h_1 \tag{B.24}$$

$$\frac{\partial p'}{\partial x} = -f_1 \quad (x = 0, L) \tag{B.25}$$

$$\begin{aligned} h_1 = &-\bar{\rho}\frac{1}{S_c}\frac{\partial}{\partial x}\left(S_c\frac{\partial \bar{u}u'}{\partial x}\right) + \frac{\bar{u}}{\bar{a}^2}\frac{\partial^2 p'}{\partial t \partial x} + \frac{\bar{\gamma}}{\bar{a}^2}\frac{\partial p'}{\partial t}\frac{1}{S_c}\frac{\partial}{\partial x}(S_c\bar{u}) \\ &- \frac{1}{S_c}\frac{\partial}{\partial x}S_c\left(\bar{\rho}u'\frac{\partial u'}{\partial x} + \rho'\frac{\partial u'}{\partial t}\right) \\ &+ \frac{1}{\bar{a}^2}\frac{\partial}{\partial x}\left(u'\frac{\partial p'}{\partial x}\right) + \frac{\bar{\gamma}}{\bar{a}^2}\frac{\partial p'}{\partial t}\frac{1}{S_c}\frac{\partial}{\partial x}(S_c u') \\ &+ \frac{1}{S_c}\frac{\partial}{\partial x}S_c(\mathcal{F}'_1 + \mathcal{F}'_{1s}) - \frac{1}{\bar{a}^2}\frac{\partial(\mathcal{P}'_1 + \mathcal{P}'_{1s})}{\partial t} \end{aligned} \tag{B.26}$$

$$f_1 = \bar{\rho}\frac{\partial u'}{\partial t} + \bar{\rho}\frac{\partial}{\partial x}(\bar{u}u') + \bar{\rho}u'\frac{\partial u'}{\partial x} + \rho'\frac{\partial u'}{\partial t} - (\mathcal{F}'_1 + \mathcal{F}'_{1s}) \tag{B.27}$$

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Linear Stability of Three-Dimensional Motions

$$k^2 - k_n^2 = -\frac{1}{\bar{a}^2}\frac{\hat{F}_n}{\hat{\eta}_n} \tag{6.89}$$

$$\begin{aligned} k^2 - k_n^2 = -i\frac{k_n}{\bar{a}E_n^2}&\left\{\oint\!\!\oint\left[\frac{\gamma\hat{\mathbf{u}}}{\hat{\eta}_n}\cdot\hat{\mathbf{n}} + \bar{\mathbf{u}}\cdot\hat{\mathbf{n}}\psi_n\right]\psi_n dS + (\gamma-1)\int\psi_n^2\nabla\cdot\bar{\mathbf{u}}dV\right\} \\ &-\frac{1}{\bar{p}E_n^2}\int\left[\frac{\hat{\mathfrak{F}}}{\hat{\eta}_n}\cdot\nabla\psi_n - i\frac{k_n}{\bar{a}}\frac{\hat{\mathfrak{P}}}{\hat{\eta}_n}\psi_n\right]dV \end{aligned} \tag{6.90}$$

$$\omega - \omega_n = \frac{1}{2}\frac{\gamma}{E_n^2}\oint\!\!\oint\frac{\hat{\mathbf{u}}^{(i)}}{\hat{\eta}_n}\cdot\hat{\mathbf{n}}\psi_n dS - \frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\int\left[\frac{\hat{\mathfrak{F}}^{(r)}}{\hat{\eta}_n}\cdot\nabla\psi_n + \frac{k_n}{\bar{a}}\frac{\hat{\mathfrak{P}}^{(i)}}{\hat{\eta}_n}\psi_n\right]dV \tag{6.93}$$

$$\begin{aligned} \alpha = -\frac{1}{2E_n^2}&\left\{\oint\!\!\oint\left[\gamma\frac{\hat{\mathbf{u}}^{(r)}}{\hat{\eta}_n}\cdot\hat{\mathbf{n}}\psi_n + (\bar{\mathbf{u}}\cdot\hat{\mathbf{n}})\psi_n^2\right]dS + (\gamma-1)\int\psi_n^2\nabla\cdot\bar{\mathbf{u}}dV\right\} + \\ &-\frac{\bar{a}^2}{2\omega_n\bar{p}E_n^2}\int\left[\frac{\hat{\mathfrak{F}}^{(i)}}{\hat{\eta}_n}\cdot\nabla\psi_n + \frac{k_n}{\bar{a}}\frac{\hat{\mathfrak{P}}^{(r)}}{\hat{\eta}_n}\psi_n\right]dV \end{aligned} \tag{6.94}$$

A Spatially Averaged Solution

$$p'(x,t) = \bar{p}\sum_{j=1}^{M}\eta_j(t)\psi_j(x) \tag{B.30}$$

$$u'(x,t) = \sum_{j=1}^{M}\frac{\dot{\eta}_j(t)}{\bar{\gamma}k_j^2}\frac{d\psi_j(x)}{dx} \tag{B.31}$$

$$\int_0^L \psi_j\psi_l S_c dx = E_l^2\delta_{jl} \tag{B.32}$$

$$E_l^2 = \int_0^L \psi_l^2 S_c dx \tag{B.33}$$

$$\frac{d^2\eta_l}{dt^2} + \omega_l^2\eta_l = F_l \tag{B.34}$$

$$F_l = -\frac{\bar{a}^2}{\bar{p}E_l^2}\left\{\int_0^L h_1\psi_l S_c dx + [f_1\psi_l S_c]_0^L\right\} \tag{B.35}$$

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Application of the General Method of Time Averaging

$$\ddot{\eta}_N + \omega_N^2 \eta_N = \mu G_N \tag{4.43}$$

In any event, for μ small, the η_N differ but little from sinusoids so (without approximation) it is reasonable to express $\eta_N(t)$ in the equivalent forms

$$\eta_N(t) = r_N(t) \sin(\omega_N t + \phi_N(t)) = A_N(t) \sin \omega_N t + B_N(t) \cos \omega_N t \tag{4.44}$$

and

$$A_N(t) = r_N \cos \phi_N \ ; \quad B_N = r_N \sin \phi_N$$

$$r_N = \sqrt{A_N^2 + B_N^2} \ ; \quad \phi_N = \tan^{-1}\left(\frac{A_N}{B_N}\right) \tag{4.45}$$

$$\mathcal{E}_N(t) = \frac{1}{2}\dot{\eta}_N^2 + \frac{1}{2}\omega_N^2 \eta_N^2 \tag{4.46}$$

$$\langle \mathcal{E}_N \rangle = \frac{1}{\tau} \int_t^{t+\tau} \mathcal{E}_N \, dt' \ ; \quad \langle \mu G_N \dot{\eta}_N \rangle = \frac{1}{\tau} \int_t^{t+\tau} \mu G_N \dot{\eta}_N \, dt' \tag{4.47}$$

Application of the General Method of Time Averaging (cont'd)

$$\frac{d}{dt}\langle \mathcal{E}_N \rangle = \mu \langle G_N \dot{\eta}_N \rangle \qquad (4.48)$$

Following Krylov and Bogoliubov (1947) we apply the 'strong' condition that the velocity is always given by the formula for an oscillator is force-free-motion,

$$\mathcal{E}_N = \frac{1}{2}\omega_N^2 r_N^2$$

$$\mu G_N \dot{\eta}_N = \mu G_N \omega_N r_N \cos(\omega_N t + \phi_N) \qquad (4.52)\text{a,b}$$

Application of the General Method of Time Averaging (cont'd)

$$\frac{\tau}{r_N}\frac{dr_N}{dt} \ll 1 \; ; \quad \frac{\tau}{2\pi}\frac{d\phi_N}{dt} \ll 1 \tag{4.53}$$

Substitution of (4.52)b in (4.48) then gives

$$\omega_N^2 r_N \frac{dr_N}{dt} = \mu \frac{\omega_N r_N}{\tau} \int_t^{t+\tau} G_N \cos(\omega_N t' + \phi_N) dt'$$

which gives

$$\frac{dr_N}{dt} = \mu \frac{1}{\omega_N \tau} \int_t^{t+\tau} G_N \cos(\omega_N t' + \phi_N) dt' \tag{4.54}$$

Because $r_N(t)$ and $\dot{r}_N(t)$ are nearly constant in the interval $(t, t+\tau)$, this relation implies the equation *before* averaging,

$$r_N \frac{d\phi_N}{dt} = -\mu \frac{1}{\omega_N \tau} \int_t^{t+\tau} G_N \sin(\omega_N t' + \phi_N) dt' \tag{4.57}$$

Application of the General Method of Time Averaging (cont'd)

With the relations (4.45), equations (4.54) and (4.57) can be converted to equations for A_N and B_N:

$$\frac{dA_N}{dt} = \frac{\mu}{\omega_N \tau} \int_t^{t+\tau} G_N \cos \omega_N t' \, dt'$$

$$\frac{dB_N}{dt} = -\frac{\mu}{\omega_N t} \int_t^{t+\tau} G_N \sin \omega_N t' \, dt' \tag{4.58)a,b}$$

These are the principal and <u>very useful</u> results of time averaging.

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\mathbf{M}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Rayleigh’s Criterion and Linear Stability

“If heat be periodically communicated to, and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged.”

Derivation

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = F_n \qquad (6.66)$$

The instantaneous energy[19] of the n^{th} oscillator is

$$\varepsilon_n = \frac{1}{2}\left(\dot{\eta}_n^2 + \omega_n^2\eta_n^2\right) \qquad (6.67)$$

and the change of energy in one cycle is the integral over one period of the rate at which work is done by the force F_n:

$$\Delta\varepsilon_n = \int_t^{t+\tau_n} F_n(t')\dot{\eta}_n(t')dt' \qquad (6.68)$$

$$\begin{aligned}
\omega^2 &= \omega_n^2 - Re\left(\frac{\hat{F}_n}{\hat{\eta}_n}\right) = \omega_n^2 - \left|\frac{\hat{F}_n}{\hat{\eta}_n}\right|\cos\phi_F \\
\alpha_n &= \frac{-1}{2\omega_n} Im\left(\frac{\hat{F}_n}{\hat{\eta}_n}\right) = \frac{-1}{2\omega_n}\left|\frac{\hat{F}_n}{\hat{\eta}_n}\right|\sin\phi_F
\end{aligned} \qquad (6.70)\text{a,b}$$

Derivation (cont'd)

The real part of F_n is

$$Re(F_n) = |\hat{F}_n| \cos(\omega_n t + \phi_F) = |\hat{F}_n| \{\cos \omega_n t \cos \phi_F - \sin \omega_n t \sin \phi_F\} \tag{6.72}$$

Hence the right-hand side of (6.68) is

$$\begin{aligned} \Delta\varepsilon_n &= \int_t^{t+\tau_n} Re(F_n) Re(\eta_n) dt' = \omega |\hat{F}_n| \int_t^{t+\tau_n} \left\{ \sin^2 \omega_n t' \sin \phi_F - \frac{1}{2} \sin 2\omega_n t' \cos \phi_F \right\} dt' \\ &= \omega |\hat{F}_n| |\hat{\eta}_n| \frac{\tau_n}{2} \sin \phi_F \end{aligned}$$

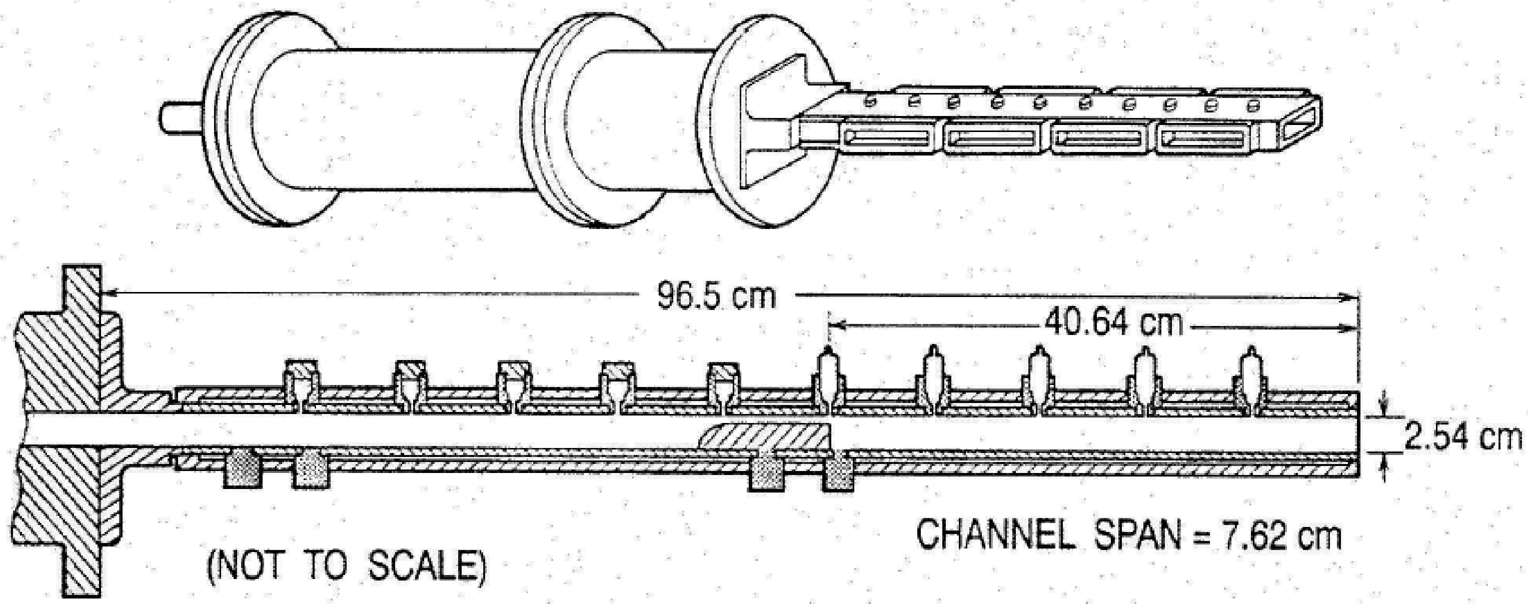
Substitution of (6.33)b leads to the formula

$$\Delta\varepsilon_n = 2\pi \alpha_n \omega_n |\hat{\eta}_n|^2 \tag{6.73}$$

<u>Conclusion</u>

Rayleigh's Criterion is equivalent to criterion for linear stability <u>when all gains and losses are accounted for.</u>

Caltech Dump Combustor



Remarks:

(i) Positive α_n (the system is linearly unstable) implies that the average energy of the oscillator increases, and vice-versa.

(ii) Rayleigh's original criterion is equivalent to the principle of linear instability **if** only heat exchange is accounted for and is **neither** a necessary **nor** a sufficient condition for existence of a combustion instability.

(iii) The extended form (6.73) of Rayleigh's Criterion is exactly equivalent to the principle of linear instability, all linear processes being accounted for.

Experimental Confirmation

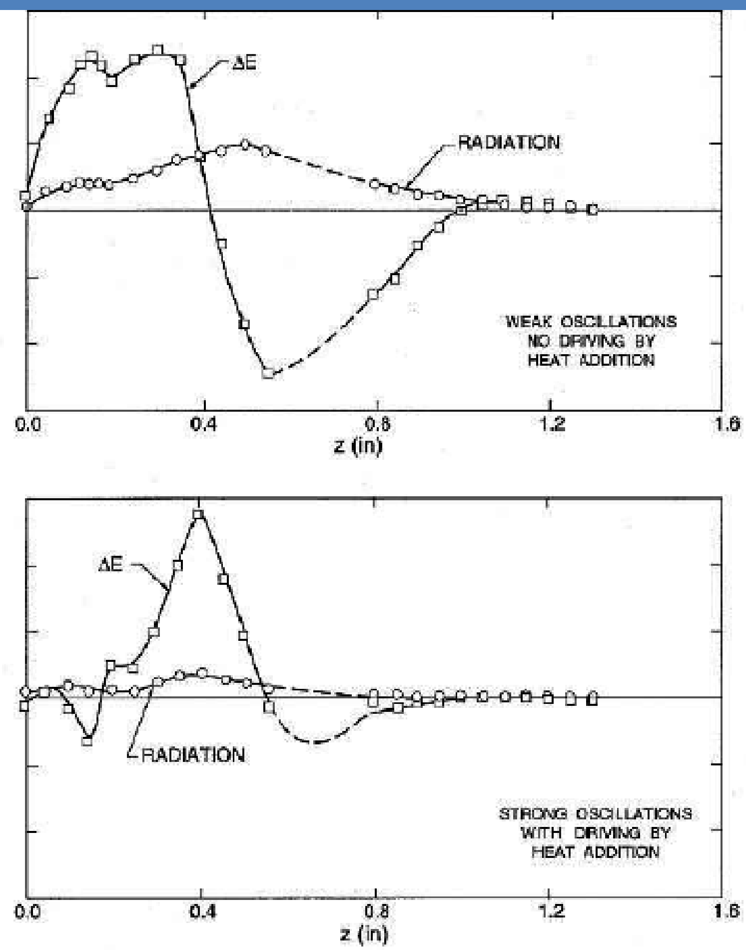


FIGURE 6.11. Experimental confirmation of Rayleigh's Criterion. Data obtained from chemiluminescence of OH (Sterling and Zukoski, 1991).

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Representation of Active Control

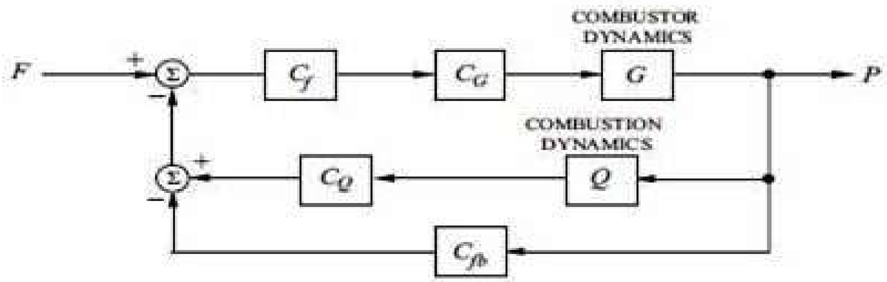


FIGURE 9.1. Block diagram for a system containing passive (C_G and C_Q) and feedback (C_f and C_{fb}) control.

The scalar transfer function P/F for the system is[3]

$$\frac{P}{F} = \frac{G(C_G C_f)}{1 + G(C_G C_f)(C_Q Q + C_{fb})} \tag{9.1}$$

[3]Although the use of block diagrams can be helpful in analysis of nonlinear systems, Figure 9.1 and the following manipulations are restricted to a linear system, here having single-input and single-output, a SISO system.

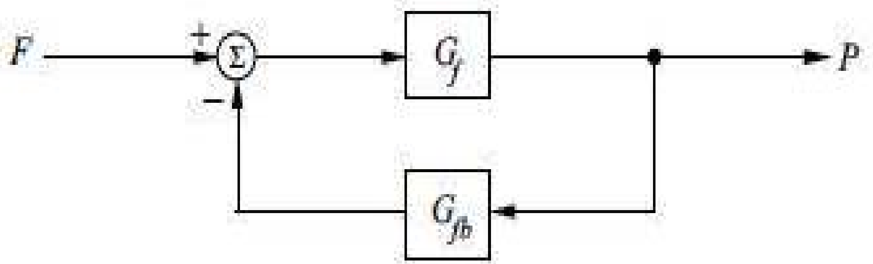


FIGURE 9.2. General block diagram of a combustion system with passive and active control.

‘General’ Block Diagram

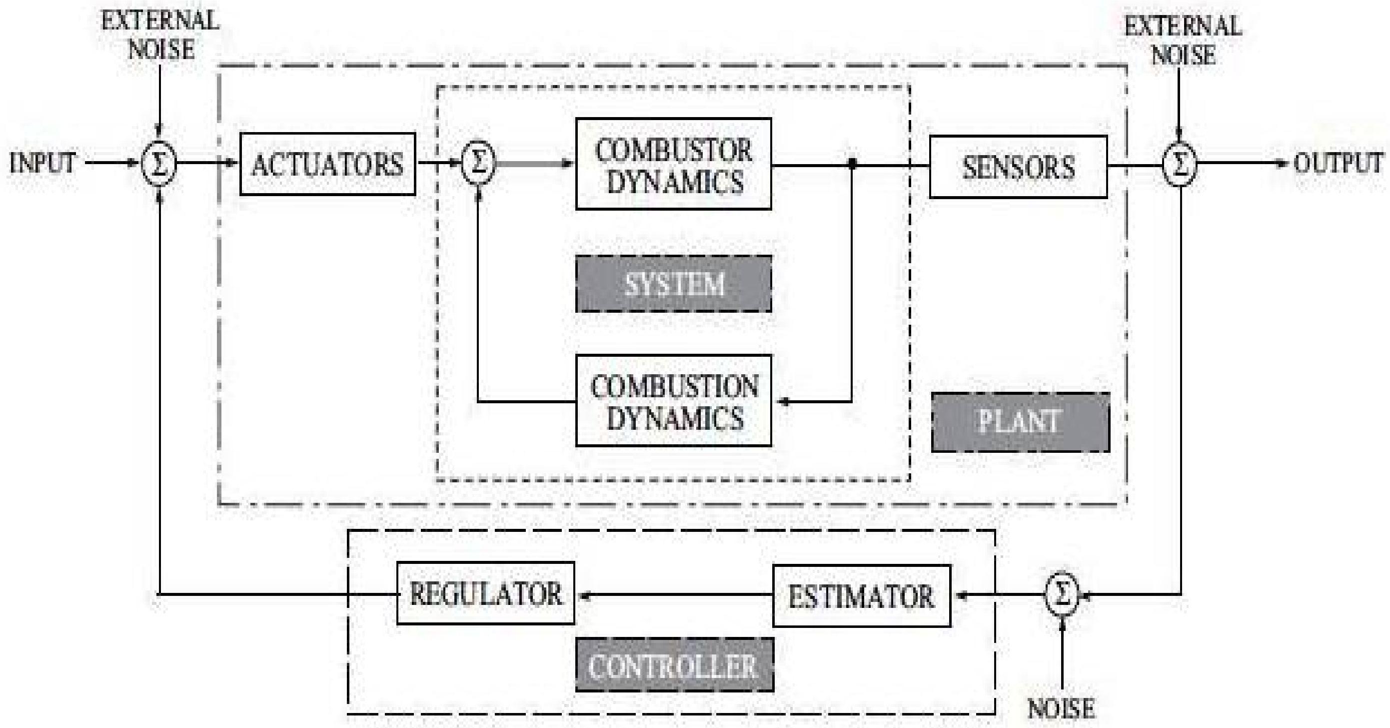


FIGURE 9.3. A general block diagram for classical and modern control.[5] (Adapted from a diagram due to Professor R.C. Murray, private communication.)

Original Proposal of Active Control

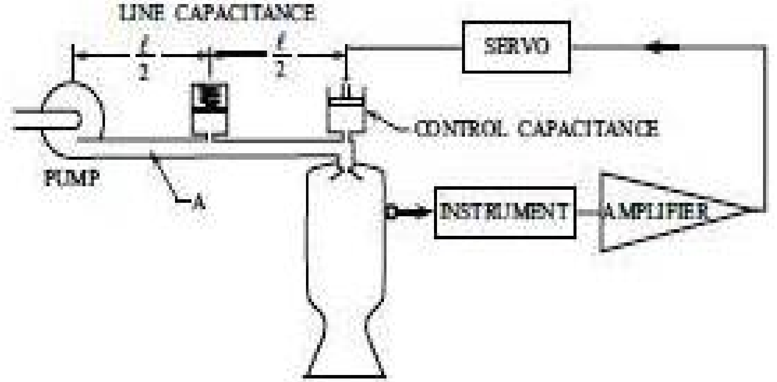


FIGURE 9.4. Schematic of the first proposal for active feedback control of the dynamics in a combustion system (Tsien 1952).

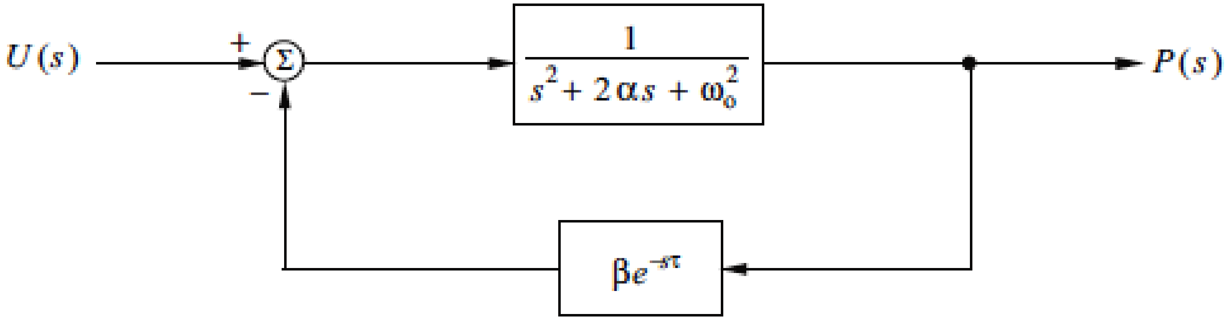


FIGURE 9.5. Block diagram for the system shown in Figure 9.4.

First Use of Fuel for Active Control

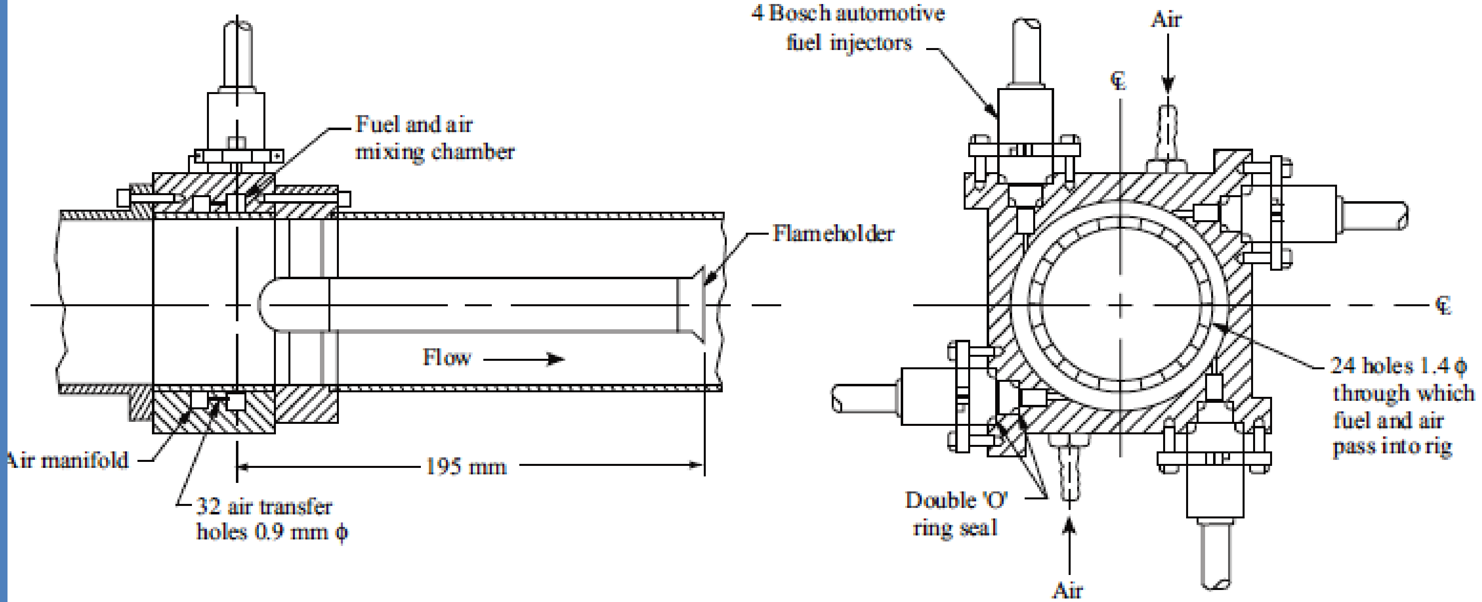


FIGURE 9.12. The device for mixing secondary fuel and air at the upstream end of the flameholder shown in Figure 9.9 (Langhorne *et al.* 1989).

Effect of Control on Combustion

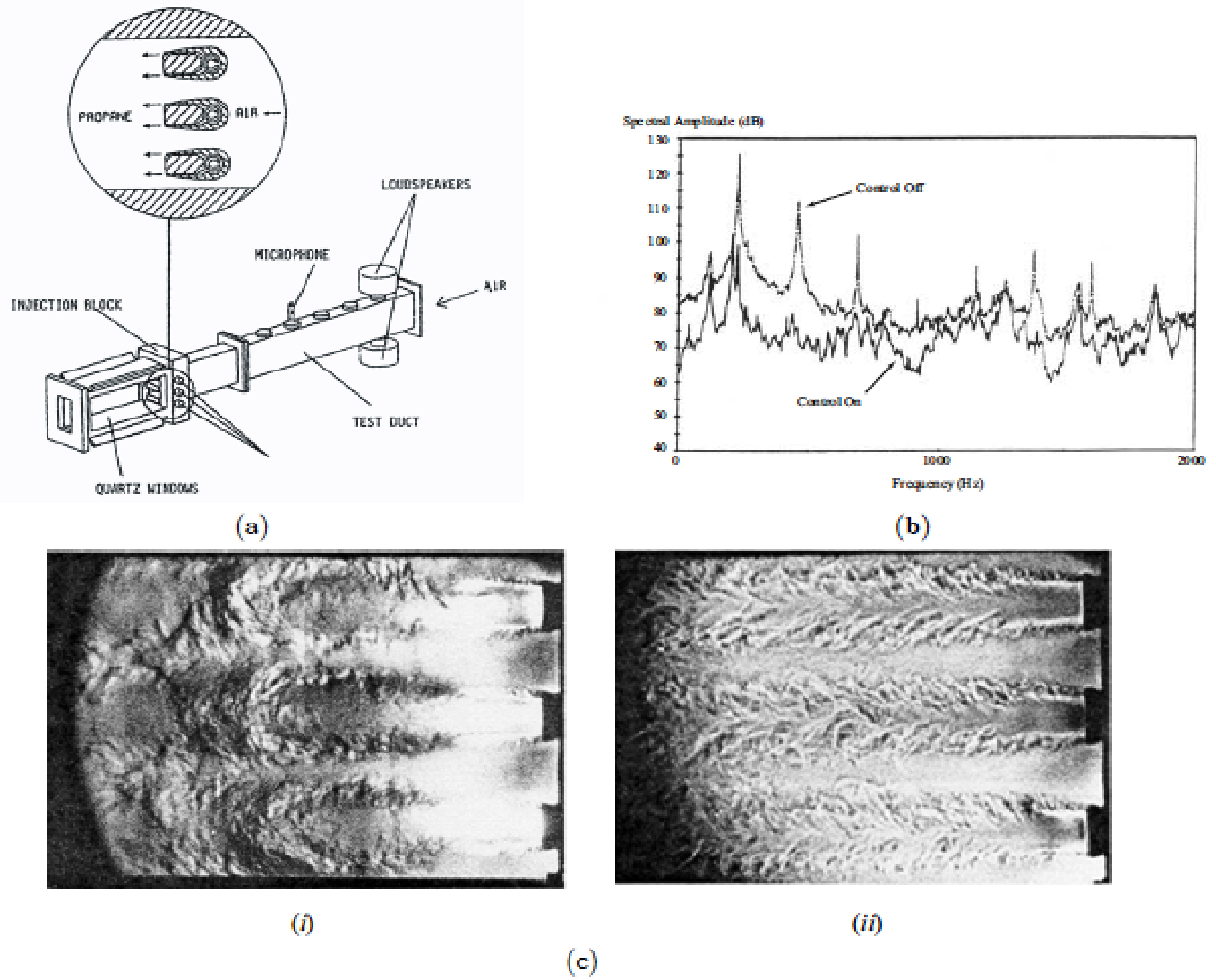


FIGURE 9.14. Feedback control at École Centrale (a) apparatus with a 250 kW combustor; (b) effects of control on the light emission; (c) schlieren photographs of the combustion region (*i*) without and (*ii*) with control (Poinsot *et al.*, 1987, 1988, 1989).

Early TUM Feedback Control of a Liquid Fuel/Air System

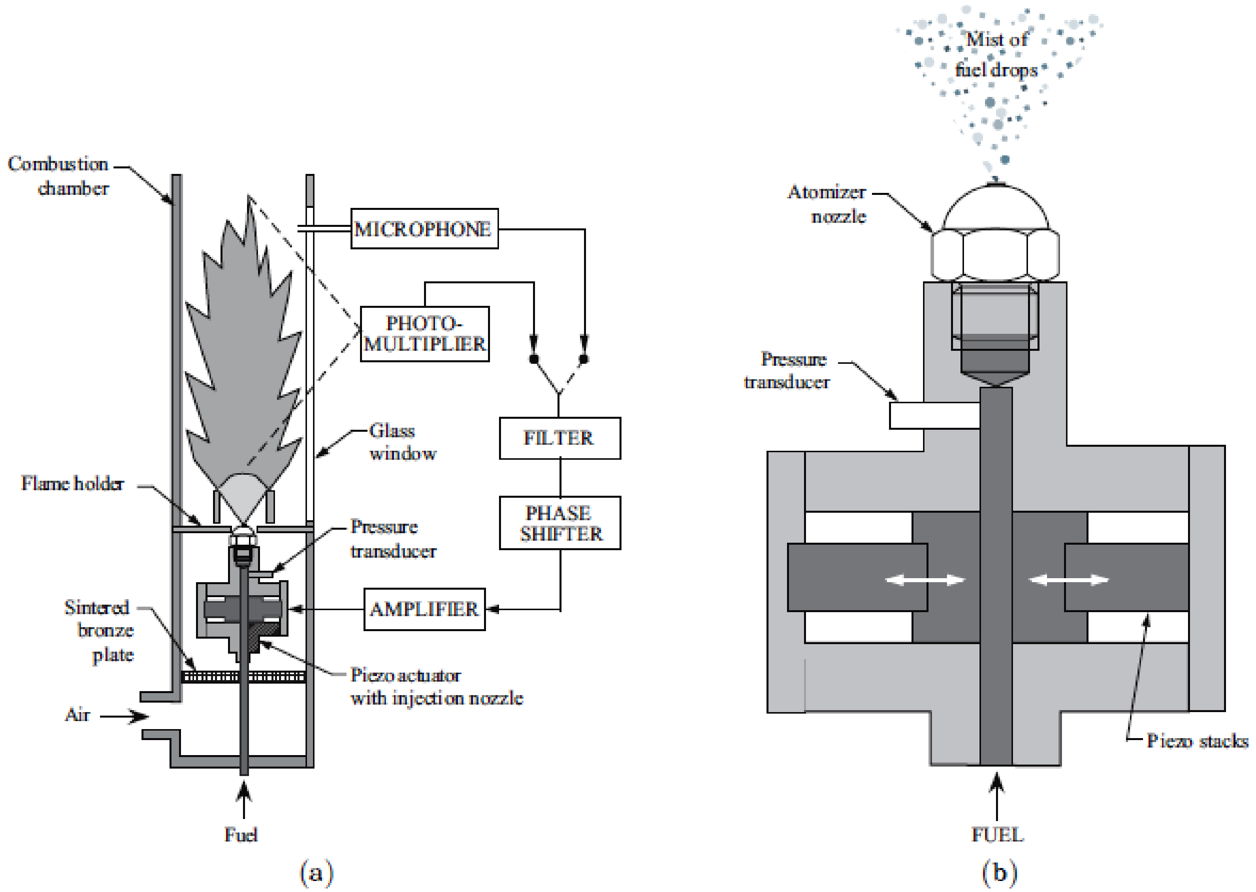


FIGURE 9.24. Feedback control at Technische Universität München. (a) apparatus; (b) the piezo actuator (Hermann, Gleis and Vortmeyer 1996).

Test Result for TUM System

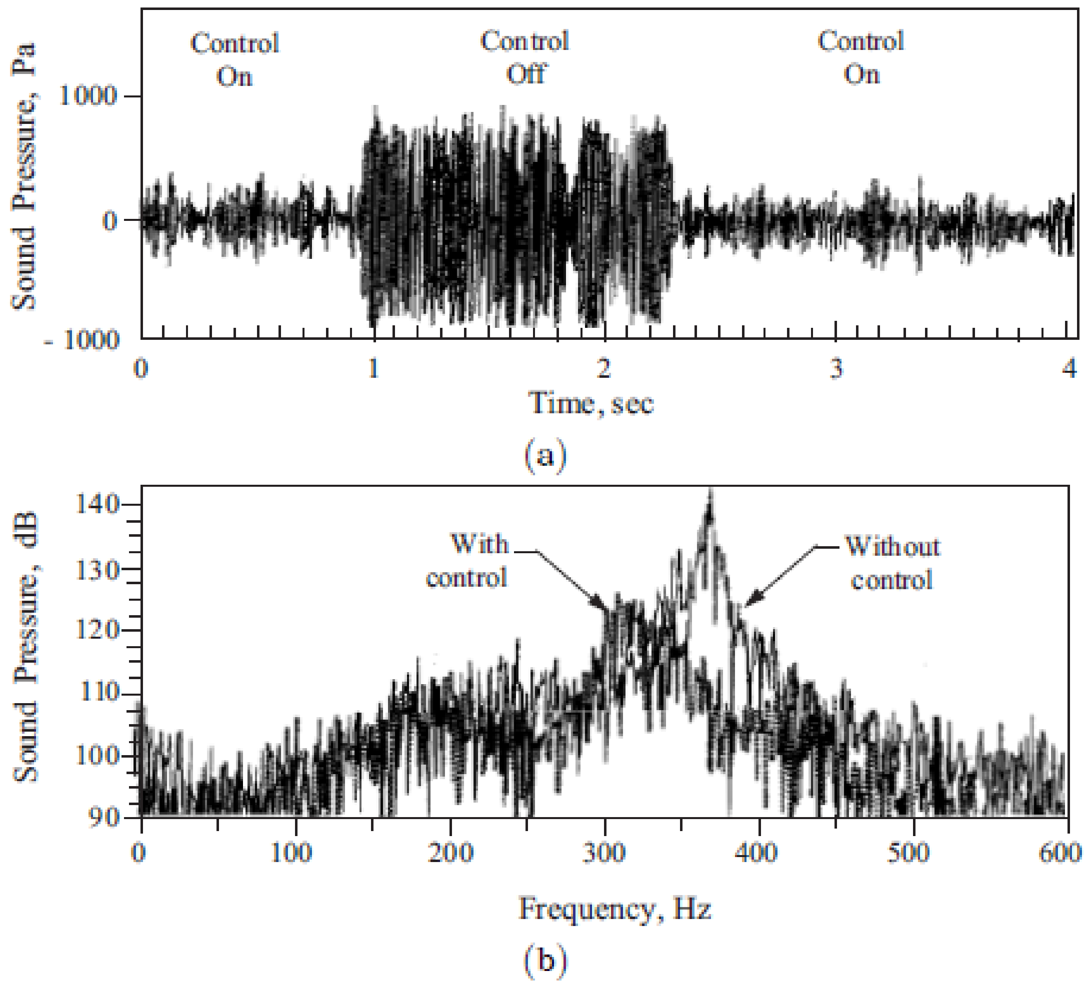
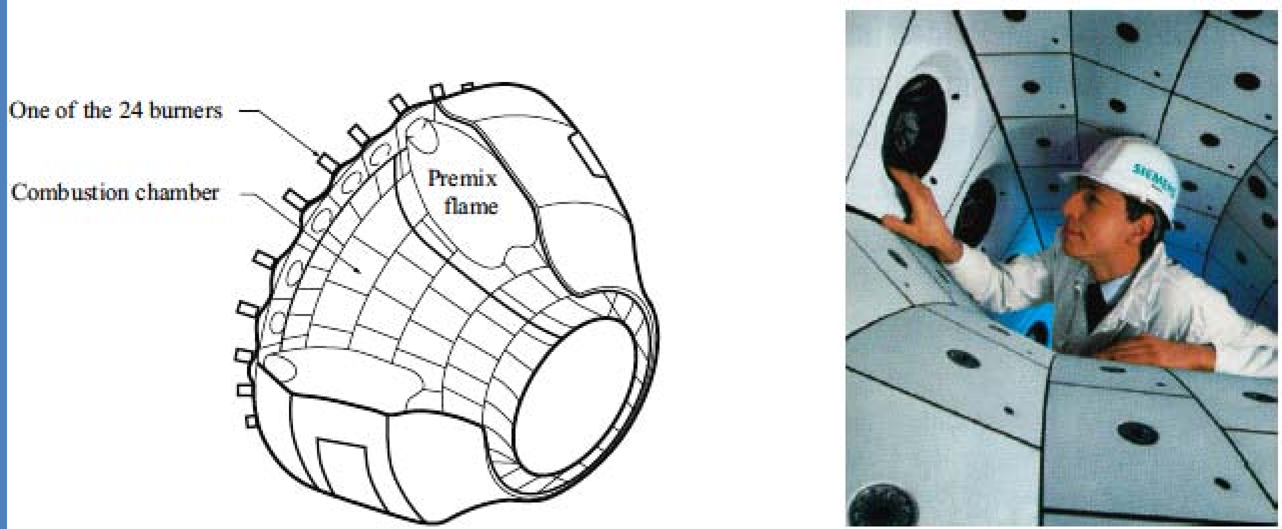


FIGURE 9.25. Test results found with the apparatus shown in Figure 9.24, Figure 14 of Hermann, Gleis and Vortmeyer (1996).

Siemens’ Experience I



(a) (b)

FIGURE 9.27. A sketch (a) of a combustor used in the Vx4.3A series Siemens gas turbines, (b) an illustration of the size of the combustor (courtesy of Siemens AG, and Dr. J. Hermann).

Siemens' Experience II

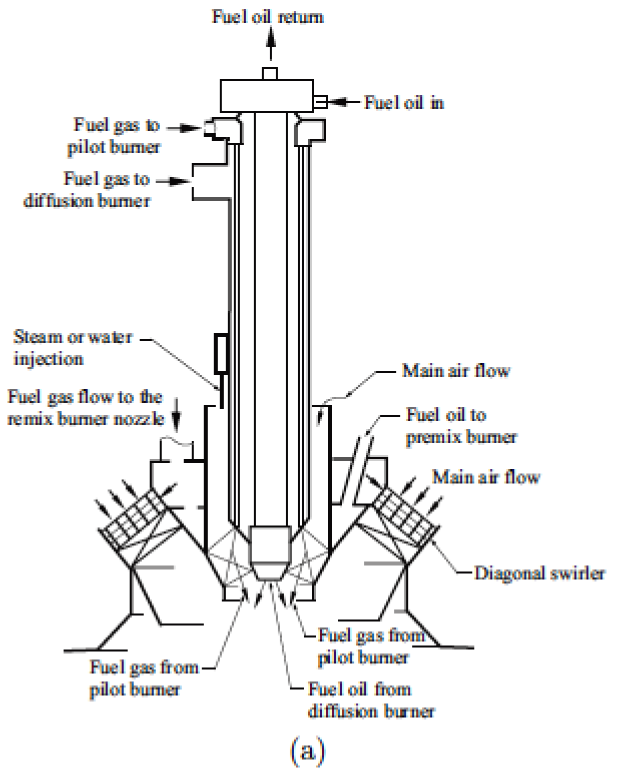


FIGURE 9.28. Simplified sketches (a) of the Siemens Hybrid Burner; (b) installation of the controller in the supply for the pilot burner (adapted from Berenbrink and Hoffmann 2000 and Hermann and Hoffmann 2005).

Rolls-Royce Annular Combustor

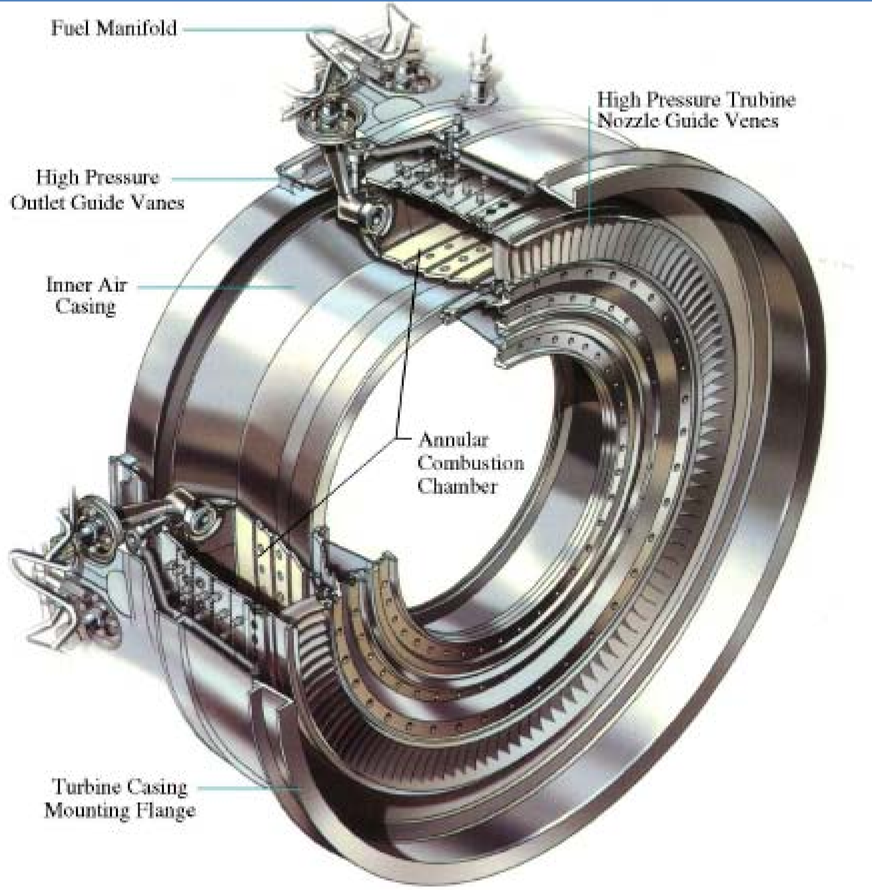
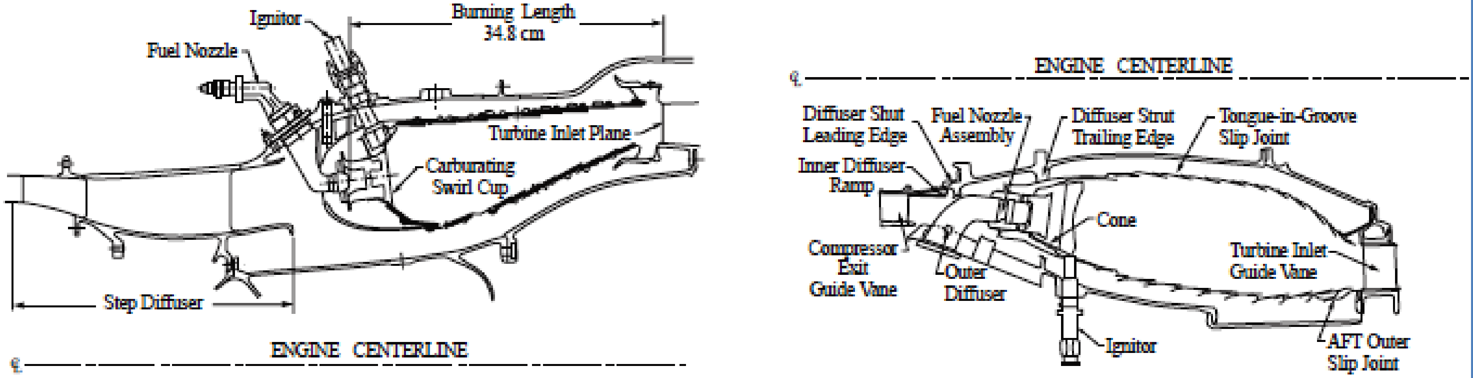


FIGURE 9.34. A sketch of an annular combustor. (Rolls-Royce 2005)

GE and P&W Combustors Prior to ECCP (1975)



(a) General Electric CF6-50 (b) Pratt & Whitney JT9D-7A

FIGURE 9.33. The two combustors chosen at the beginning of the Experimental Clean Combustor Program (ECCP). There is no significance to the different relative locations of the engine centerlines.

GE and P&W Combustors Developed in ECCP

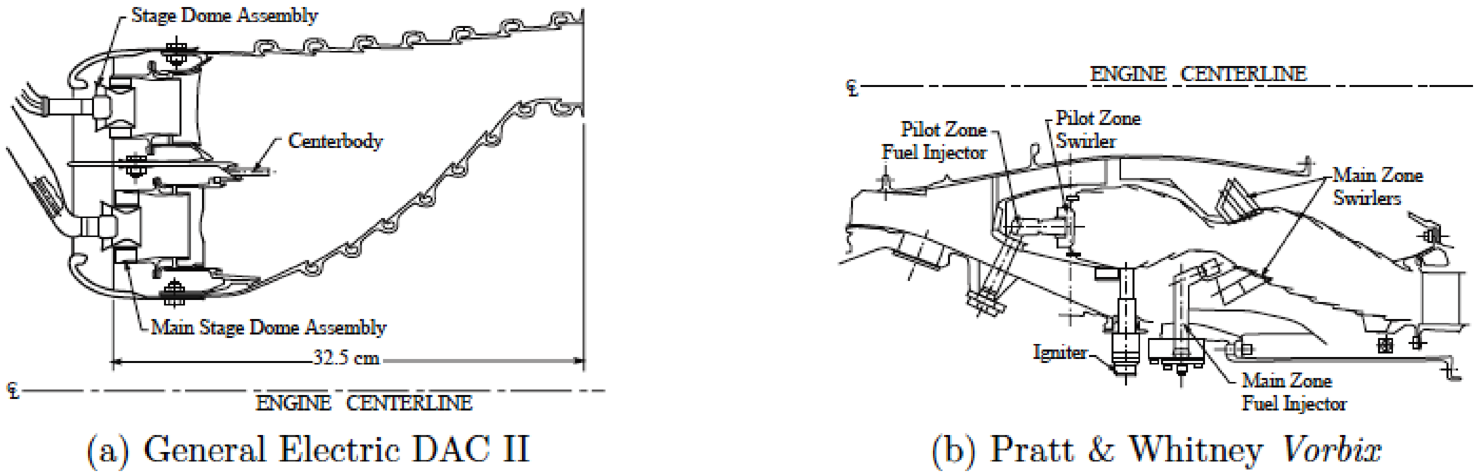


FIGURE 9.35. The GE and PW combustor designs at the end of the Experimental Clean Combustor Program (ECCP), c. 1977. (Gleason and Bahr 1979; Robers, Fiorentino, and Greene 1977)

Pratt and Whitney Talon Designs (c. 1980-2007)

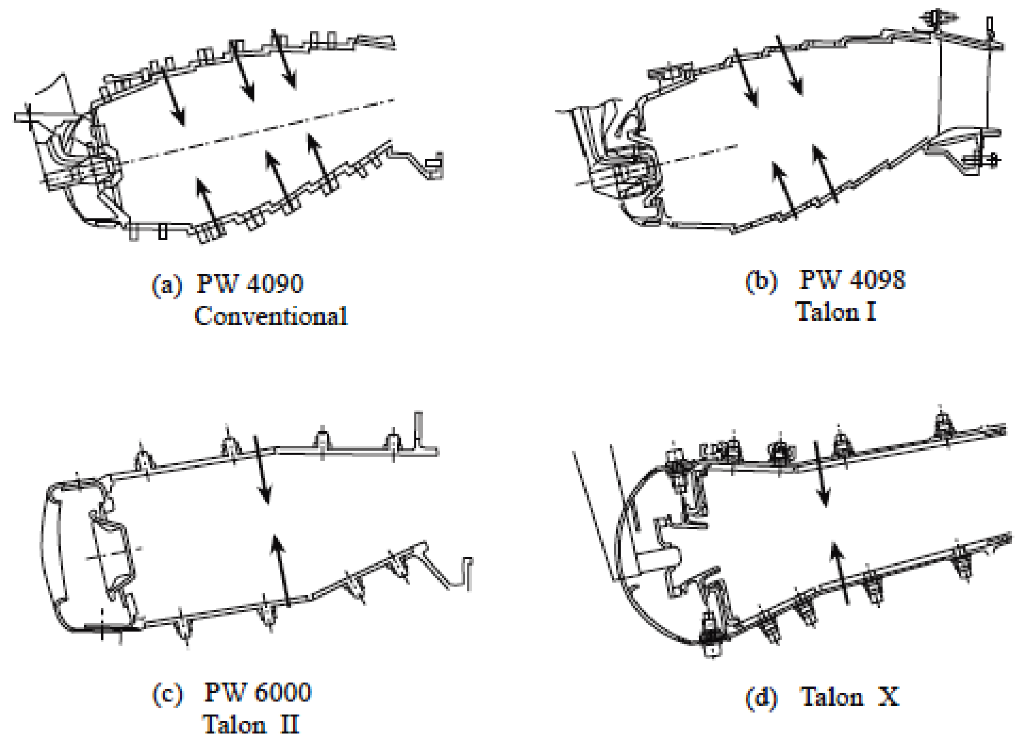


FIGURE 9.36. The sequence of four TALON designs. (McKinney 2007)

Schematic of the Flow Field in the Talon X Combustor

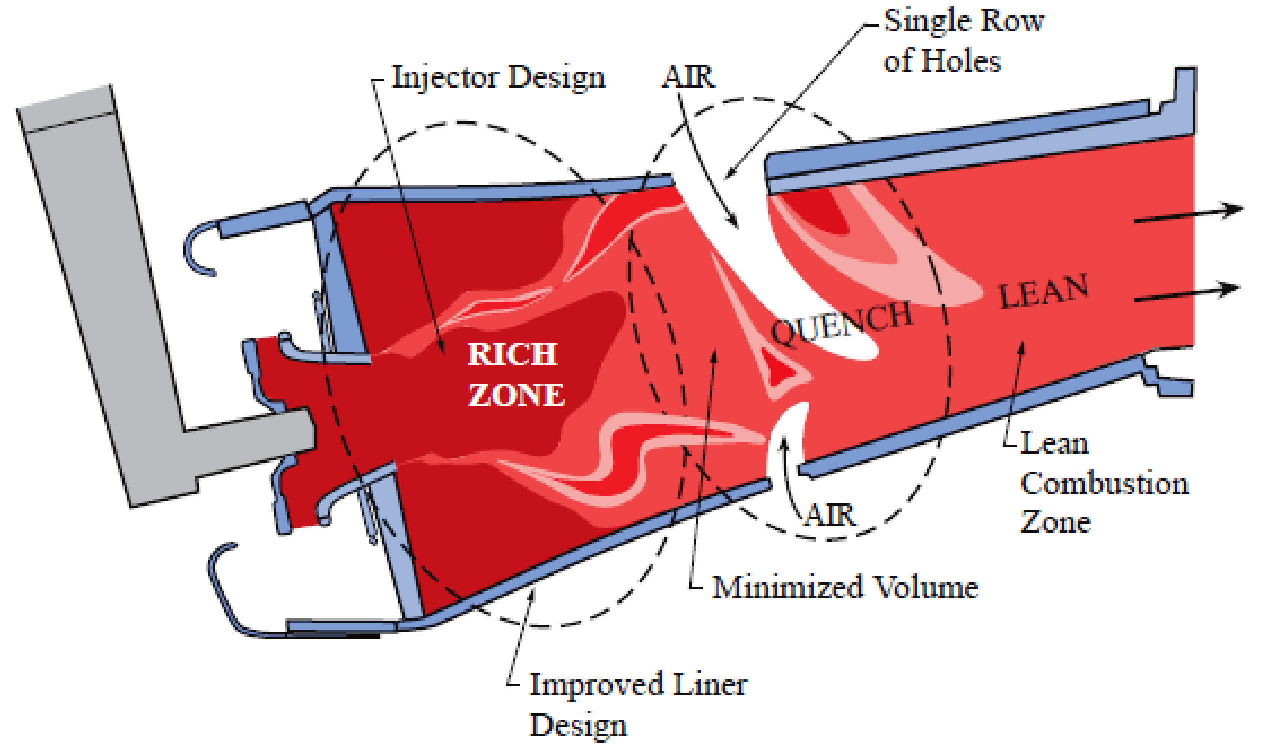


FIGURE 9.37. Section of the TALON X combustor. (McKinney 2008)

Schematic of Approximate Analyses

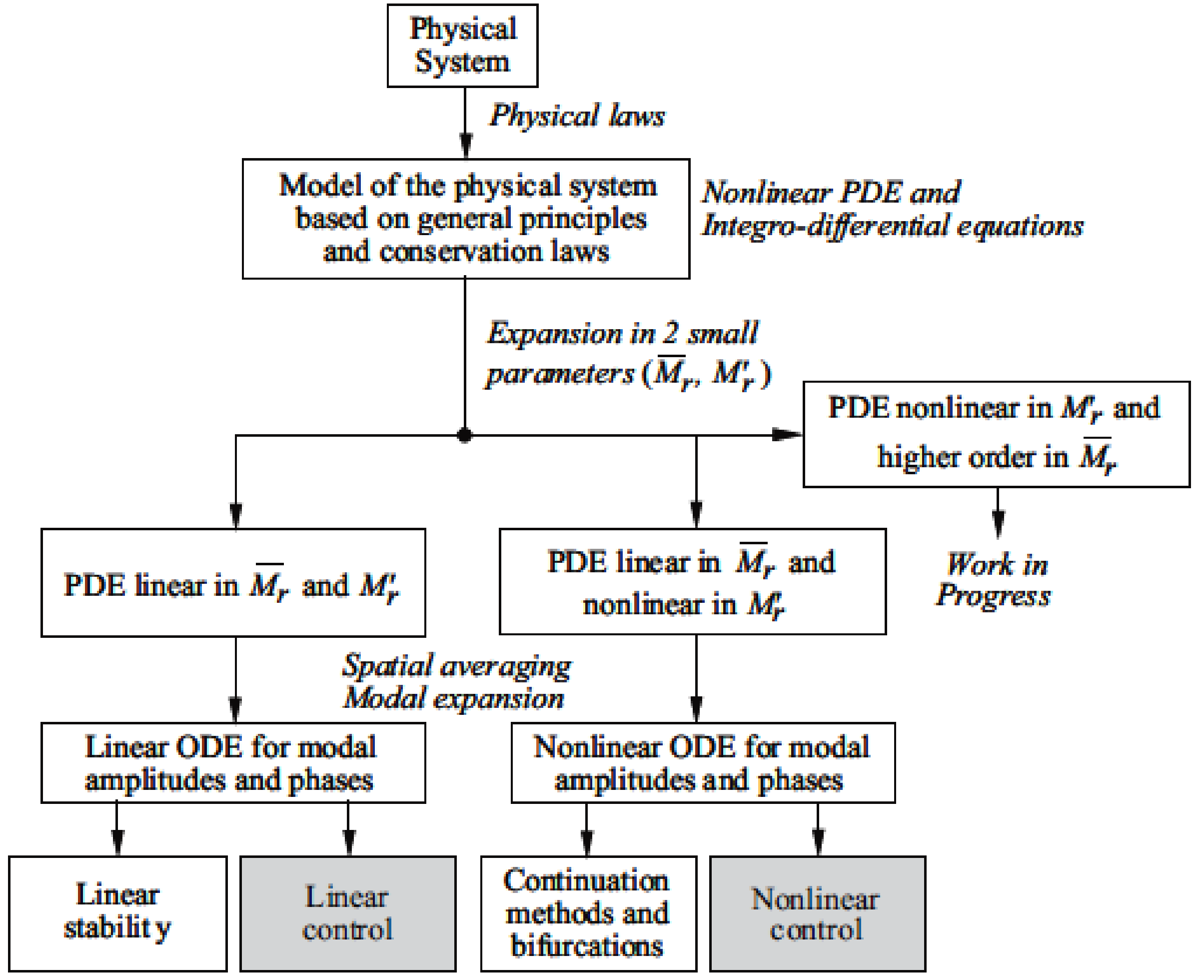


FIGURE 9.19. The general scheme according to the procedures followed here for connecting the physical system (a combustor), physical modeling, mathematical modeling, dynamics and control.

I Introduction

II Chief Mechanisms of CI and Unsteady Motions

III A Framework of Analysis Based on Spatial Averaging

IV Equations for One-Dimensional Unsteady Flow

V Results for p' and $\dot{\mathbf{M}}'$, Linear Motions

VI Time Averaging

VII Rayleigh's Criterion and Linear Stability

VIII The Case for Active Control in Liquid-fueled Systems

IX Concluding Remarks

Concluding Remarks (I)

1) $\frac{P'}{\bar{p}_r}$ & $\bar{M}_r$ 'small' implies restrictions on the expansion of the PDE.
2) Problems to be treated are dominated by wave motions

PRIMARY

3) Spatial averaging built on expansions in eigenfunctions for unperturbed problems with homogeneous B.C. (e.g. rigid walls)
4) The perturbation/iteration procedure produces results satisfying the actual B.C. to the order of the expansions.

SECONDARY

5) The formulation allows treatment of steady waves and 'general' time-dependent motions.
6) Time-averaging may be used to reduce N second order inhomogeneous equations to 2N first order equations valid for 'slow' changes of amplitudes and phases (VERY USEFUL)

Concluding Remarks (II)

7) Eigenfunctions calculated for actual problems (i.e. (3) plus perturbations) are non-orthogonal. Hence the solutions computed with the perturbation/iteration procedure are non-normal in the current jargon. Simplest realistic case is linear steady waves:

$$\hat{p}^{(1)}(\mathbf{r}) = \psi_N(\mathbf{r}) + \sum_{n=0}^{\infty}{}' \frac{\psi_n(\mathbf{r})}{E_n^2\,(k^2 - k_n^2)} \left\{ \iiint_V \psi_n(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_n(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.82}$$

$$i\bar{\rho}\bar{a}k\mathbf{M}'^{(1)} = -\nabla\hat{p}^{(1)} - \varepsilon\mu\{[\hat{\mathbf{M}}]\}_1 + \varepsilon\hat{\mathfrak{F}}_{10} + \varepsilon\mu\hat{\mathfrak{F}}_{11} \tag{4.85}$$

$$k^2 = k_N^2 + \frac{1}{E_N^2} \left\{ \iiint_V \psi_N(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_N(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.19}$$

8) Many results of the ‘standard’ analysis based on spatial averaging have compared well with experimental results (e.g. instability, particle damping,…)

9) The outstanding current deficiency of results based on non-normality is the absence of quantitative comparisons with alternative analyses and experimental results.