0.1 Use $E = h\nu$; $h = 6.626 \times 10^{-34}$ J s; $\nu = 1/T$ [T: period]

(a) $E = (6.626 \times 10^{-34}\ \mathrm{J\,s})/(1.0 \times 10^{-15}\ \mathrm{s}) = \underline{6.626 \times 10^{-19}\ \mathrm{J}}$

(b) $E = h/(1.0 \times 10^{-14}\ \mathrm{s}) = \underline{6.626 \times 10^{-20}\ \mathrm{J}}$

(c) $E = h/(1.0\ \mathrm{s}) = \underline{6.626 \times 10^{-34}\ \mathrm{J}}$

0.2

$$\rho = \left(\frac{8\pi hc}{\lambda^5}\right)\frac{\mathrm{e}^{-hc/\lambda kT}}{1-\mathrm{e}^{-hc/\lambda kT}} \text{ [eqn 0.4]}$$

$$= \frac{(8\pi hc/\lambda^5)}{\mathrm{e}^{hc/\lambda kT}-1}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}\lambda} = -\frac{40\pi hc/\lambda^6}{\mathrm{e}^{hc/\lambda kT}-1} + \frac{(8\pi hc/\lambda^5)(hc/\lambda^2 kT)\mathrm{e}^{hc/\lambda kT}}{(\mathrm{e}^{hc/\lambda kT}-1)^2} = 0$$

That is, at the maximum

$$\frac{5}{\lambda} = \frac{(hc/\lambda^2 kT)\mathrm{e}^{hc/\lambda kT}}{\mathrm{e}^{hc/\lambda kT}-1} = \frac{(hc/\lambda^2 kT)}{1-\mathrm{e}^{-hc/\lambda kT}}$$

and hence

$$\frac{hc}{5\lambda kT} = 1-\mathrm{e}^{-hc/\lambda kT}$$

At short wavelengths ($hc/\lambda kT \gg 1$)

$$\frac{hc}{5\lambda kT} \approx 1, \text{which implies that } \underline{\lambda T \approx hc/5k}$$

Exercise: Confirm that the extremum of ρ is in fact a maximum.

0.12 Use the experimental data at 195 nm and eqn 0.6 to compute the work function of the metal surface.

$$\begin{aligned}\Phi &= h\nu - E_{\mathrm{K}} = (hc/\lambda) - \tfrac{1}{2}m_{\mathrm{e}}v^2 \\ &= (6.626\times10^{-34}\ \mathrm{J\,s})(3.00\times10^{8}\ \mathrm{m\,s^{-1}})/(195\times10^{-9}\ \mathrm{m}) - \\ &\quad \tfrac{1}{2}(9.10938\times10^{-31}\ \mathrm{kg})(1.23\times10^{6}\ \mathrm{m\,s^{-1}})^2 \\ &= 3.303\times10^{-19}\ \mathrm{J}\end{aligned}$$

When light of wavelength 255 nm is used, the kinetic energy of the ejected electron is

$$\begin{aligned}E_{\mathrm{K}} &= (hc/\lambda) - \Phi \\ &= (6.626\times10^{-34}\ \mathrm{J\,s})(3.00\times10^{8}\ \mathrm{m\,s^{-1}})/(255\times10^{-9}\ \mathrm{m}) - \\ &\quad 3.303\times10^{-19}\ \mathrm{J} \\ &= 4.492\times10^{-19}\ \mathrm{J}\end{aligned}$$

corresponding to a speed of

$$v = \left(\frac{2E_{\mathrm{K}}}{m_{\mathrm{e}}}\right)^{1/2} = \underline{9.93\times10^{5}\ \mathrm{m\,s^{-1}}}$$

Exercise: For the above problem, what is the longest wavelength of light capable of ejecting electrons from the metal surface?

0.17 First, equate the Coulombic force of attraction between the electron and the nucleus with the centrifugal force due to the circular motion of the electron:

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

where v is the velocity of the electron. Since the angular momentum of the electron is an integral multiple of $\hbar$,

$$v = \frac{n\hbar}{m_e r}$$

so that

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{n^2\hbar^2}{m_e r^3}$$

or

$$r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}$$

The total energy E is the sum of the kinetic and potential energy. Use of the expressions given above for v and for r yields

$$\begin{aligned} E &= \tfrac{1}{2}m_e v^2 - \frac{e^2}{4\pi\varepsilon_0 r} \\ &= -\frac{e^4 m_e}{32n^2\varepsilon_0^2\hbar^2\pi^2} \end{aligned}$$

which, upon replacing m_e by μ and $\hbar$ by $h/2\pi$ results in eqn 0.12.