the fractional fluctuation is $\Delta T/T = \sigma_{sky}/2.7$ as shown in Table 2.

Lasenby and Davies[9] also point out that overestimation of the errors of the data points will bias the estimates of $\hat{\sigma}$ downward. Thus, they suggest adopting the larger of two other statistical measures c and s_{ii}^2 as a more conservative upper limit on $\hat{\sigma}^2$. The first involves a test of the hypothesis that $\sigma_{sky} > 0$ against $\sigma_{sky} = 0$; if $\hat{\sigma}^2 > c$, we infer $\sigma_{sky} > 0$. The entries in Table 2, row 2 are for a test at the 95% significance level. The second quantity is a 95% upper confidence level on $\hat{\sigma}$ (see Table 2, row 3) and is related to the statistic used by Boynton and Partridge[15,16] which produces similar limits on σ_{sky} for our data.

In the case of the Norwegian data, the value of $\Delta T/T$ calculated from $\hat{\sigma}^2$ is slightly larger than that found from the measure c, suggesting the possibility that real fluctuations are present. As this result is not confirmed in the Italian data, and is barely significant, we prefer to treat our data as upper limits on $\Delta T/T$ on angular scales of ~2-3°. Using the combined data set, we have $\Delta T/T \leq 5.6 \times 10^{-4}$ at 95% confidence limits.

Table 2 Upper limits at 95% confidence on $\Delta T/T = \sigma_{sky}/2.7$ determined from the observations using the statistical measures of ref. 9

	$\Delta T/T$ ($\times 10^{-4}$)			
			Combined data	
Statistical measure used	Norwegian data	Italian data	Unsmoothed $n=1$ ($\theta \approx 2°$-$3°$)	Smoothed $n=2$ ($\theta \approx 5°$)
$\hat{\sigma}^2$	3.6	−2.4	3.2	3.5
c	3.5	3.0	3.1	3.0
s_{ii}^2	6.2	1.6	5.6	6.8

We may also smooth our observations by taking a running average of pairs of points (Table 2). Finally, note that for comparison with theoretical predictions of the angular correlation function of temperature fluctuations, the appropriate angular scale is 10°, our beam-switching angle.

Although not as sensitive as some results reported from shorter wavelength or larger-scale measurements[4,5,7], our results show either that some causal process acted to smooth the Universe on large scales before the epoch of last scattering (as in the inflationary models), or that the matter distribution in the Universe was very homogeneous *ab initio.* For instance, for large-scale adiabatic[1,2] perturbations, we expect $\Delta T/T \sim 1/3\, \Delta\rho/\rho$; hence our results imply $\Delta\rho/\rho \lesssim 2 \times 10^{-3}$ for density perturbations at $z \sim 1{,}000$.

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Is there an unusual solar core?

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Various attempts have been made in the past two decades to determine the properties of the core of the Sun, by measurements of solar neutrino radiation, the distortion of the solar gravitational potential and frequency splittings of solar oscillation modes. In the latter category, the two best measurements have been made by Duvall and Harvey[1] and by Brown[2]; both showed splitting roughly independent of spherical harmonic degree *l*, and both had a peculiar peak in the measured splitting at $l = 11$. We present here new results, based on the analysis of 6,656 individual oscillation modes for $5 \leq l \leq 20$. These data yield a splitting spectrum which is consistent with previous measurements, but without the unusual peak at $l = 11$, thus suggesting that a simple standard model for the solar core is essentially correct.

Several recent measurements have been interpreted as evidence for an unusual solar core, where unusual here means either rotating rapidly relative to the Sun's surface or containing very strong magnetic fields.

The first accurate measurement of the shape of the Sun in 1966 by Dicke and Goldenberg[3] yielded an oblateness[4] of $\Delta r \equiv r_{equator} - r_{pole} = 41.9 \pm 3.3$ arc ms (or a solar gravitational quadrupole moment $J_2 \times 10^6 = 23.7 \pm 2.3$), much larger than the $\Delta r = 7.8$ arc ms caused by surface rotation alone. Further analysis of the same data revealed a 12.4-day rotating distortion which was interpreted as the signature of a rotating magnetic core in the Sun[5]. However, the measurement was repeated[6,7] in 1983 and a much smaller oblateness of $\Delta r = 15.4 \pm 4.1$ arc ms was found (after taking into account any possible latitude-dependent brightness signal[6]), giving $J_2 \times 10^6 = 5.3 \pm 2.8$, and no significant 12-day oscillations were observed. Analysis of new 1984 data from the distortion telescope has yielded similar results, with a smaller oblateness measured for that year[23].

Claverie *et al.*[8] detected a 12.6-day oscillation in the line-of-sight velocity of the integrated solar surface, which was interpreted as evidence of a rotating magnetic core. However, the observations were later found to be explained by sunspots travelling across the solar disk[9-11].

A triplet structure with a spacing of 0.75 μHz was found in $l = 1$ p-modes by the Birmingham group[12], which was initially taken to be rotational[12] or magnetic[13-15] splitting of the different m-states of the $l = 1$ modes. However, it now seems that the fine structure in the power spectrum could be a manifestation of the short lifetimes of the p-modes, and not a true splitting[16-18]. Analysis of 290 days of solar irradiance data collected by the Solar Maximum Mission spacecraft[16,17] indicated that linewidths for $l = 0, 1, 2$ modes are typically 1.5 μHz, and a 2.5σ upper limit of 1.0 μHz was found for the splitting between adjacent m-values of $l = 1$ modes from line-broadening measurements. This implies that $J_2 \times 10^6 < 1.0$ from rotation.

Hill[19] recently measured p-mode splittings of 1.8 ± 0.1 μHz for $1 \leq l \leq 6$ modes, which is greater than other measurements and is especially inconsistent with the results of Duvall and Harvey and the present work. One possible explanation for the discrepancy may lie in Hill's data set, which measured only three solar diameters on the Sun at each time interval and is therefore sensitive to all modes with $l < 50$ (ref. 19). That proper mode identification has been made for the $\sim 10^4$ modes with frequencies from 2 to 4 mHz in a single power spectrum has yet to be established.

Duvall and Harvey[1] measured the splitting of $m = \pm l$ p-modes for $1 \leq l \leq 100$, finding a nearly constant splitting of 0.46 ± 0.02 μHz. The splitting decreased slightly with decreasing *l*, with a peak at $l = 11$, then turned over at $l \approx 4$ to further increase with decreasing l, but with quite large errors for smaller l. Inversion of this splitting spectrum[20] gave a solar internal rotation that is approximately constant with radius down to $r = 0.3\ R_{\odot}$, giving $J_2 \times 10^6 = 0.17 \pm 0.04$. Note that this value of J_2 is calculated from a derived rotation versus depth curve, unlike the direct measurement of J_2 from the solar shape measurement[3]. It is still possible that a large magnetic field may distort the mass distribution of the solar interior, giving a larger J_2, although how such a field would be stabilized is uncertain.

Brown[2] recently made another measurement of the splitting for modes of all m-states with $8 \leq l \leq 50$, finding splittings slightly larger than the Duvall and Harvey numbers for $10 < l < 30$, and also nearly exactly the same peak at $l = 11$. Neither peak by itself could be claimed significant, but the appearance of a single nearly identical peak in two different data sets was suggestive of an unusual solar core[2].

The splitting measurements by Duvall and Harvey used measured individual mode frequencies to determine splittings, but their observational technique of optically averaging the solar image allowed some leakage of modes with $m < l$ into their data, which affected the derived splittings. Problems of this sort were considered in their analysis[1]. Brown used full two-dimensional spatial filtering to isolate modes, which virtually eliminates this problem from the outset. However, the many modes considered in that analysis necessitated a simple cross-correlation approach to measure the splittings[2]. I have found that such an approach is a poor substitute for the use of isolated individual mode frequencies. To address better the question of the splitting of low-l oscillation modes, I have investigated the 6,656 individual modes for all m-values with $5 \leq l \leq 20$ and having radial order $9 \leq n \leq 24$.

In 1984-85, a new telescope was built at Big Bear Solar Observatory, and is now dedicated to helioseismology observations. Using a Zeiss 0.25-Å bandpass birefringent filter chopped ±0.125 Å by an electro-optical crystal, solar images in the two wings of the 6,439-Å Ca line were alternately formed on an RCA Newvicon TV camera. A COSMOS image processor digitized and added/subtracted the blue wing/red wing images at the rate of 7.5 pairs per s, and added 375 pairs to produce one 192×240-pixel Doppler image of the Sun each minute. For our present analysis, data from 12 contiguous days of observations, 22 July 1985 to 2 August 1985, were scaled, rotated and collapsed to form 8,150 32×32-pixel images. After subtracting a running mean image to remove slow trends, the residuals were fitted to individual spherical harmonics with $5 \leq l \leq 20$ and the resulting 416 time series of fit coefficients were padded with zeros and Fourier-transformed.

Individual modes were easily identified using existing tables[21], and the mode frequencies were fit to a polynomial in l and n as a first step in the analysis. Using the fit polynomial as a guide, gaussian curves were fit to the individual modes and the l–n polynomial fit was redetermined, leaving residuals of ±2 μHz. Mode frequencies were further refined by then refitting the gaussians and by using a CLEAN algorithm[1] on 10-μHz-wide segments of the power spectra centred on the fit frequencies. The mode frequencies determined from both these techniques were combined, and each was assigned a weight using its fit gaussian width, the second moment of its CLEANed spectrum, and the ratio of power in centred 10- and 50-μHz sections of its CLEANed spectrum. The mode frequencies for each l were then fit by weighted least-squares to

$$\nu_{nlm} \rightarrow R_{nl} - A_l m + B_l m^2 - C_l m^3$$

Figure 1 shows the results of the fit, which are in good agreement with Brown's[2] data. The larger noise at small l is caused by the decreasing total number of modes available for the fit; all three plots are consistent with there being no l-dependence in the data. For comparison with both Brown and Duvall–Harvey the data have been redisplayed in Fig. 2 as the rotational splitting plotted against l. Here, it is clearly seen that the 30-nHz peak at $l = 11$ shown by both previous splitting measurements is not visible in the Big Bear data set. Assuming that no real peak exists in the splitting spectrum, one can conclude from these results and the above-mentioned results of other solar researchers that there is little or no evidence at present for a rapidly rotating and/or magnetic solar core.

The Big Bear data also show the solar differential rotation (Fig. 3). This plot and the cubic fit coefficients C_l in Fig. 1 are in good agreement with Brown's results.

The measured splitting in Fig. 2 is roughly constant at 450 nHz, which is ~10 nHz larger than the same region measured

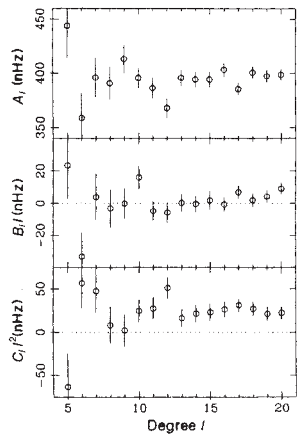

Fig. 1 Linear, quadratic and cubic fit coefficients described in the text. Factors of l and l^2 have been included to remove most of the l-dependence. The fits were done from a synodic reference frame.

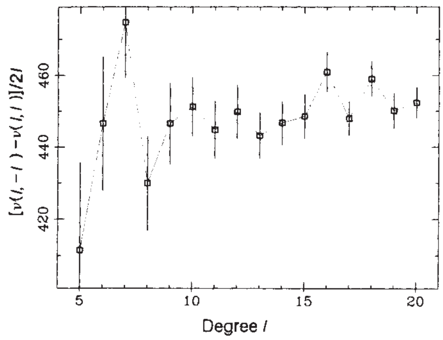

Fig. 2 Rotational splitting of oscillation modes plotted against degree, averaged over radial order n for each l. $A_l + C_l l^2 + 31$ is plotted from the fit, where the 31 nHz is added to shift to a sidereal reference frame.

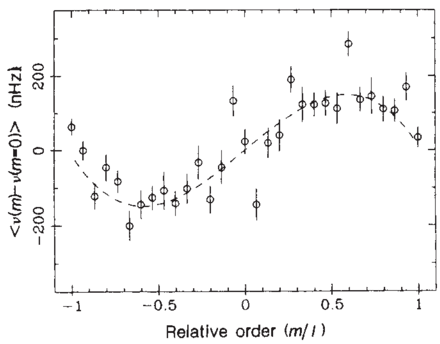

Fig. 3 Dependence of oscillation frequency on order m, where all modes with $10 \leq l \leq 20$ and $9 \leq n \leq 24$ have been combined, and a linear dependence of $420m$ nHz (synodic) has been subtracted. Dashed line, best-fit cubic $\nu(m) = 381(m/l) - 369(m/l)^3$.

by Duvall and Harvey. This may be an artefact of the different methods of data analysis, or some small mode leakage in the Duvall-Harvey data[1]; or it may be a physically significant shift. Howard[22] used 62 yr of Mt Wilson sunspot data to measure averaged systematic changes of ~5 nHz in the Sun's surface rotation rate at different times in the solar cycle. We may now be seeing a related shift in 3 yr of helioseismology data.

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Mode decoupling during retrorefraction as an explanation for bizarre radar echoes from icy moons

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Radar echoes from Europa, Ganymede and Callisto, large ice-covered moons of Jupiter, have characteristics that differ markedly from those of other natural targets[1-3]. A decoupling of two characteristic modes of propagation appears to be the key to an understanding of the anomalous distribution of power among the polarized components of the echoes. Such decoupling is inherent in the double-reflection theory of Ostro and Pettengill[4] and the subsurface scattering model of Goldstein and Green[2]. However, neither of these theories explains the astounding strengths of the echoes as well as the retrorefraction peak in the refraction scattering model of Hagfors *et al.*[5], which in turn is less able to represent the power distribution. The two concepts of mode decoupling and retrorefraction are combined here to explain the strength, distribution and other features of the echoes. Causes of the decoupling are proposed and their morphological implications are introduced.

Important aspects of the problem are illustrated in Table 1. The first column gives the polarized and normalized radar cross-sections measured at wavelength 0.13 m for Europa, from the review by Ostro[6]. In each case a polarized wave, either linear (L) or circular (C), is transmitted from Earth and echo power is measured in the same (S) and orthogonal (O) senses of polarization. Note how the numbers for Europa compare with the next column of representative values for rocky planets and our Moon, the echoing properties of which are relatively well understood. Particularly striking are the Europa cross-sections which exceed those of a perfectly reflecting metal sphere, and the enormous returns in the 'unexpected' OL and SC polarizations where smooth reflecting surfaces would give zero values. These are the principal features to be explained by theory.

In their treatment of refraction scattering, Hagfors *et al.*[5] envisage a large number of centres of refraction beneath or on the surface, away from which the refractive index decreases to merge with a smaller background value representing the bulk of the near-surface material of the moon. Radar ray paths are conducted around or bent toward these centres as in a lens, and those rays that are deflected by π radians make up the echoes that are received back on Earth. Hagfors *et al.*[5] use a geometrical optics (GO) treatment for spherical lenses which predicts infinite retrorefraction with the echo power distributed among polarizations, as given in Table 1 (GO sphere). This model is successful in illustrating strong echoes and the large OL and SC components in particular. The authors invoke more standard scattering mechanisms to explain differences between theory and observation for the relative strengths of the returns. For example, they must ascribe all of the observed OC echo to such sources. However, anomalously large cross-sections are observed for all polarizations. If this model can produce only three large values, there remains the need for a second special source for the fourth. The quest here is for a single explanation of all four.

First consider the meaning of the GO infinities: they can be removed by the quasi-wave-optical (WO) method that has been used for the forward caustics in radio occultations by planetary atmospheres[7] and gravitational lenses[8]. For retrorefraction this yields

$$\sigma/\pi a^2 = (16\pi a/\lambda q)P \qquad (1)$$

where σ is the total radar cross-section; a the ray impact parameter which is the same as the radius of the coherent aperture and is assumed to be large relative to the wavelength λ; q is a numeric defined at $\alpha = \pi$ by $|da/d\alpha| = a/\pi q$ where α is the angle of refraction; and $0 \leq P \leq 1$ is a polarization factor to be discussed. Equation (1) corresponds to antenna-like radiation from the coherent annular aperture shown in Fig. 1*A* having radius a and strip width $(\lambda a/\pi q)^{1/2}$. The (a/λ) factor in equation (1) shows promise for explaining strong echoes, if P is not too small.

But that is a problem. Figure 1*A* illustrates that $P = 0$ for all retrorefracting lenses that are perfect spheres; that is, a polarized incident wave emerges from the coherent annular aperture with components that exactly cancel each other. Thus, the infinities of GO become zeros of the quasi-WO solution. The theory appears to be as bizarre as the observations. However, $P = 4\sqrt{2}/\pi^2$ at $\alpha = \pi - \lambda/2a$ so there is a cone of strong refraction with a null at $\alpha = \pi$. If the lenses were imperfect, the null could be filled with energy from this adjacent conical lobe.

For an imperfect spherical or spheroidal lens, the aperture of Fig. 1*A* is no longer coherent. Ray paths emerging at substantially different azimuths are of randomly different lengths so the annulus breaks up into local patches or glints of coherence, as illustrated in Fig. 1*B*. If there were more than about 2π patches, the polarization would be essentially constant over their individual areas and the echo power from the individual glints would be summed. It follows that for a loss-less spheroidal lens

$$\sigma/\pi a^2 = T = (16F/q)(H/\lambda) \qquad (2)$$

where H is the circumferential length of a representative coherent patch and F the fraction of azimuths covered by such patches. With these concepts, $F < 1$ and $\lambda < H < a$. Consideration of the polarization of the refracted rays, as in Fig. 1*B*, yields the 'WO spheroid' column of Table 1. This evaluates the GO infinities but still leaves the zero for the OC echo.

In finding echo polarizations in the above example, each incident ray was broken down into two modes, a transverse electric (TE) mode (electric vector normal to the plane of the refracted ray in the lens) and a transverse magnetic (TM) mode (electric vector in the plane of propagation). It was assumed that these modes were affected in the same way by the lens, except for the geometrical reversal of the electric field of the TM mode, before they were reassembled at the lens exit into